Thermodynamics -1

MEP 112

2026

CONTENTS

MEP 112 Thermodynamics 1

3 Cr. hrs. = [2 Lect. + 2 Tut + 1 Lab]

Introductory comments - concepts and Definition - Heat and work .Properties of pure Substances .The First Law of Thermodynamics and its applications to control Mass and control volume. The Second Law of Thermodynamics .Carnot Cycle – Entropy. Applications of the Second Law of Thermodynamics to the Control of mass and volume– Ideal Gases .– Some Power cycles (Simple steam ., refrigeration ., and gas Cycle).

مكق١١٢ ديناميكا حرارية١

مقدمة–تعاريف–الشغل و الحرارة–خصائص المادة النقية–القانون الاول للديناميكا الحرارية و تطبيقاته على النظام المغلق و المفتوح

. القانون الثانى للديناميكا الحرارية و تعريف االنتوربيا–دورة كارنوت–الغازات المثالية–دورات الغاز و البخار(دورات البخار، دورات التبريد، دورات الغاز).

Laboratory:

1. Temperature measurements.
2. Testing of a simple power cycle.
3. Testing of a simple refrigeration cycle.

Textbook:

- R. E Sonntag. C. Borgnakke & G. J van wylen, fundamentals of Thermodynamics, 2009

References:

- Yunus A. cengel and Michael A. Boles. Thermodynamics an Engineering Approach, va Nostrand

Reinhold's, 2002.

Assessment:

Final Exam: 50 % , Quizzes: 20%, Year Work: 20%, Experimental/Oral: 10%

CHAPTER 1

Introduction and Definitions

Thermodynamics

What is thermodynamics?

- The study of thermodynamics is concerned with ways energy is stored within a body and how energy transformations, which involve heat and work, may take place.

•Approaches to studying thermodynamics

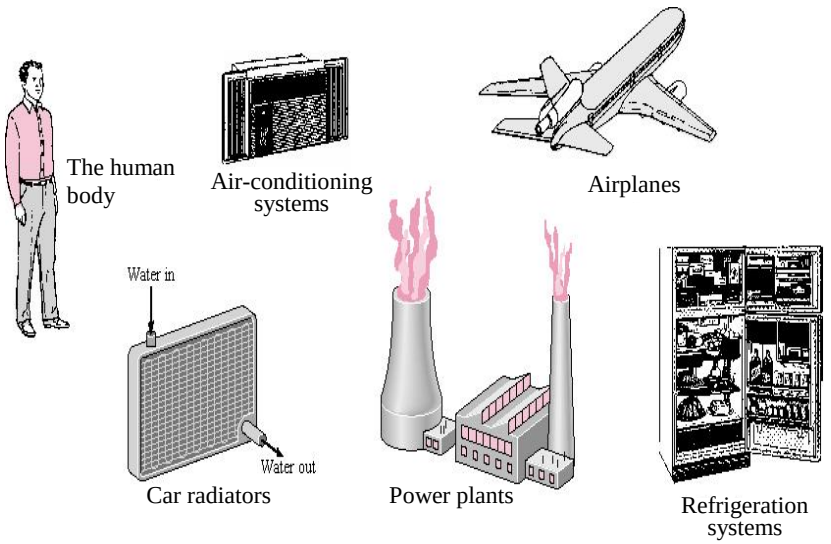
– Macroscopic (Classical thermodynamics)

- study large number of particles (molecules) that make up the substance in question
- does not require knowledge of the behavior of individual molecules

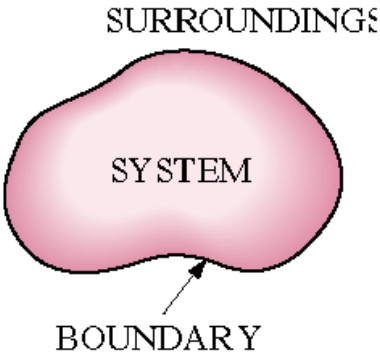
– Microscopic (Statistical thermodynamics)

- concerned within behavior of individual particles (molecules)
- study average behavior of large groups of individual particles

Applications of Thermodynamics



Thermodynamic Systems



Thermodynamic System

- quantity of matter or a region of space chosen for study

Boundary

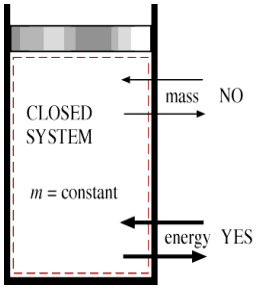
- real or imaginary layer that separates the system from its surroundings

Surroundings

- physical space outside the system boundary

Types of Systems

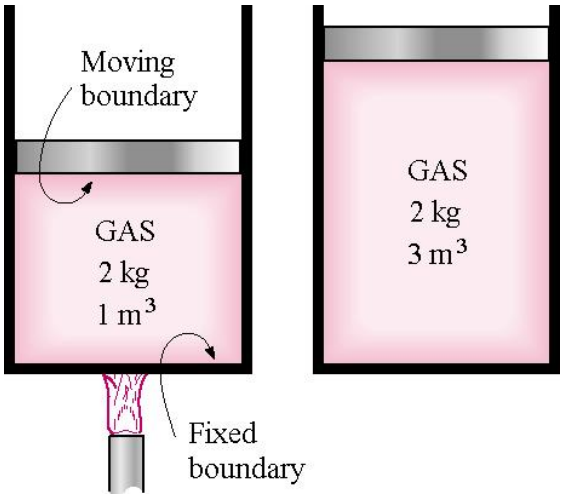
- Closed
- Open



Closed Systems (fixed masses)

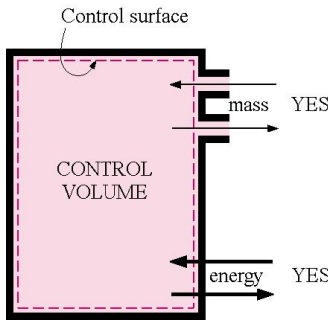
Energy, not mass, crosses closed-system boundaries

Closed System with Moving Boundary



Open Systems (Control Volumes)

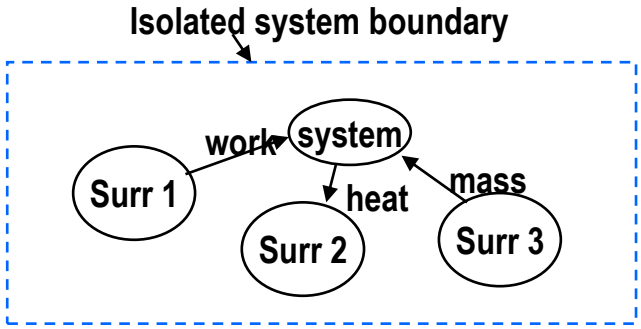
Mass and Energy Cross Control Volume Boundaries



Isolated System

Closed system where no heat or work (energy) may cross the system boundary

-typically a collection of the a main system (or several systems) and its surroundings is considered an isolated system



Total Energy of a System

- Sum of all forms of energy (i.e., thermal, mechanical, kinetic, potential, electrical, magnetic, chemical, and nuclear) that can exist in a system
- For systems we typically deal with in this course, sum of internal, kinetic, and potential energies

$$E = U + KE + PE$$

E = Total energy of system

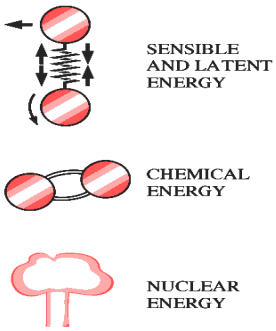
U = internal energy

KE = kinetic energy = $mV^2/2$

PE = potential energy = mgz

System's Internal Energy

System's Internal Energy = Sum of Microscopic Energies



Properties

- Any characteristic of a system in equilibrium is called a property.
- Types of properties
  - Extensive properties - vary directly with the size of the system
    Examples: volume, mass, total energy
  - Intensive properties - are independent of the size of the system
    Examples: temperature, pressure, color
- Extensive properties per unit mass are intensive properties.

specific volume v = Volume/Mass = V/m

density r = Mass/Volume = m/V

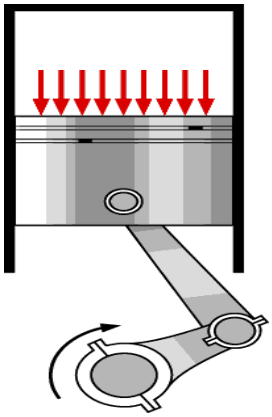
State & Equilibrium

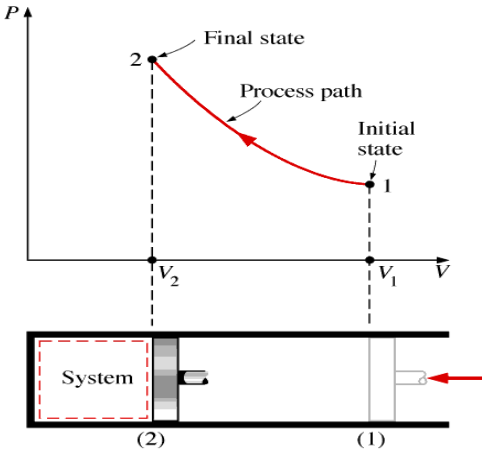
- **State of a system**
  - **system that is not undergoing any change**
  - **all properties of system are known & are not changing**
  - **if one property changes then the state of the system changes**
- **Thermodynamic equilibrium**
  - **"equilibrium" - state of balance**
  - **A system is in equilibrium if it maintains thermal (uniform temperature), mechanical (uniform pressure), phase (mass of two phases), and chemical equilibrium**

Processes & Paths

- **Process**
  - when a system changes from one equilibrium state to another one
  - some special processes:
    - isobaric process - constant pressure process
    - isothermal process - constant temperature process
    - isochoric process - constant volume process
    - isentropic process - constant entropy (Chap. 6) process
- **Path**
  - series of states which a system passes through during a process

Compression Process





Quasi-Equilibrium Processes

- System remains practically in equilibrium at all times
- Easier to analyze (equations of state can apply)
- Work-producing devices deliver the most work
- Work-consuming devices consume the least amount of work

State Postulate & Cycles

- **State Postulate**
  - The thermodynamic state of a simple compressible substance is completely specified by two independent intensive properties.
- **Cycles**
  - A process (or a series of connected processes) with identical end states

Physical properties of a system

1.Intensive properties or variables

Properties whose values don't depend on the material Present in the system.

Eg. Temperature ,Pressure, Viscosity ,Surface tension

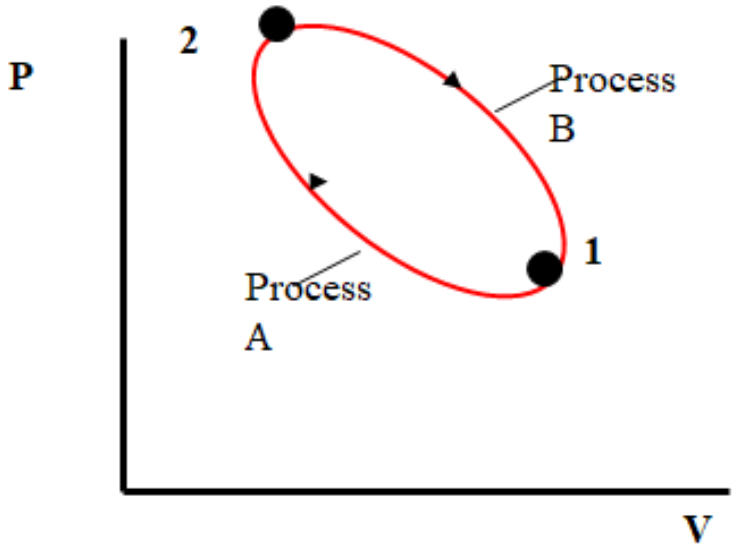
2.Extensive properties or variables.

properties whose values depend on the amount of the material present in the system.

Eg. mass, volume ,energy ,heat ,capacity ,entropy

Thermodynamic process

- cyclic process
- Reversible process
- Irreversiible process
- isothermal process
- Adiabatic process



Cyclic process:

when a system in a given initial state goes through various processes and finally return to its initial state, the system has undergone a cyclic process.

- ## Reversible process:

It is defined as a process that once having takes place it can be reversed.In doing So it leaves no change in the system or boundary.

Irreversible process:

A process that cannot return both the system and surround to their original conditions.

Adiabatic process:

A thermodynamic process during which heat energy is transferred to or from the system.

Eg: usually a fast process like filling a tank

Isothermal process:

A thermodynamic process that takes place at a constant temperature.

Eg: Usually a slow process like a baloon expanding slowly during the day

Reversible process	Irreversible process
It is a slow process going through a series of smaller stages with each stage maintaining equilibrium between the system and surroundings.	In this process the system attains final state from the initial state with a measurable speed. During the transformation, there is no equilibrium maintained between the system and surroundings.
A reversible process can be made to proceed in forward or backward direction.	Irreversible process can take place in one direction only.
The driving force for the reversible process is small since the process proceeds in smaller steps.	There is a definite driving force required for the progress of the irreversible process.
Work done in a reversible process is greater than the corresponding work done in irreversible process.	Work done in a irreversible process is always lower than the same kind of work done in a reversible process.
A reversible process can be brought back to the initial state without making an change in the adjacent surroundings.	An irreversible process cannot be brought back to its initial state without making a change in the surroundings.

state functions:

state functions are variables which are determined only by the initial and final states of the system and not by the path followed during the change from initial to the final state.

Eg: Internal energy(U), enthalpy(H), entropy (S),free energy (G)

Path functions:

Path functions are quantities that depend on the path followed during a change.

Eg: Heat absorbed (Q), Work done (w).

Concept of heat &Work

Heat:

Heat is defined as a quantity that flows across the boundary of a system during a change in its state by virtue of a difference in temperature between the system and surroundings and flows from a point of higher temperature to a point of lower temperature

Work:

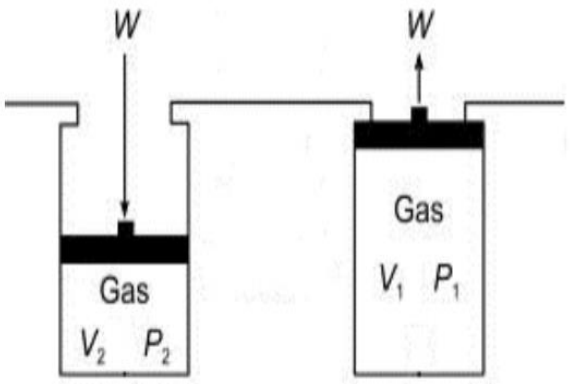
Work is defined as any quantity that flows across the boundary of a system during a change in its state and completely convertible into the lifting of a weight in the surroundings.

Work of expansion:

work done during the expansion of a gas is called work of expansion.

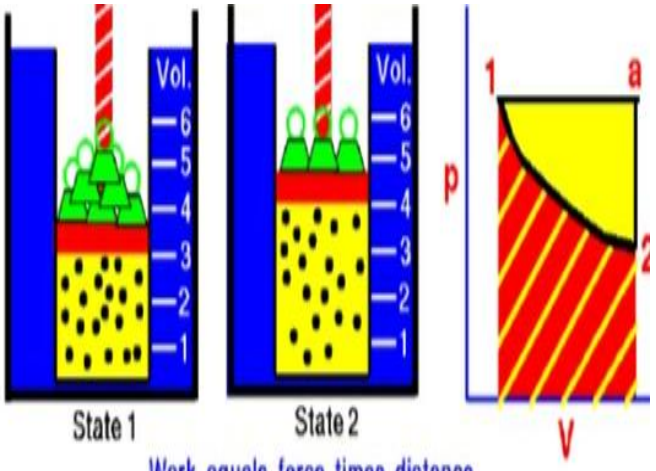
Work of compression:

The workdone by the surroundings to compress the gas is known a work of compression



Compression
Surroundings do work on system

Expansion
System does work on surroundings



Work equals force times distance.

For a gas: $W = p\,V$ Units: $\frac{\text{force}}{\text{area}}$ volume = force length

Pressure and volume vary: $W = \int dW = \int_1^2 p\,dV$

Amount of work depends on initial and final state and the path.

Properties of Pure Substances

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A Pure Substance

Is a substance that is chemically homogenous and fixed in chemical composition.(e.g. water, nitrogen, air & etc.)

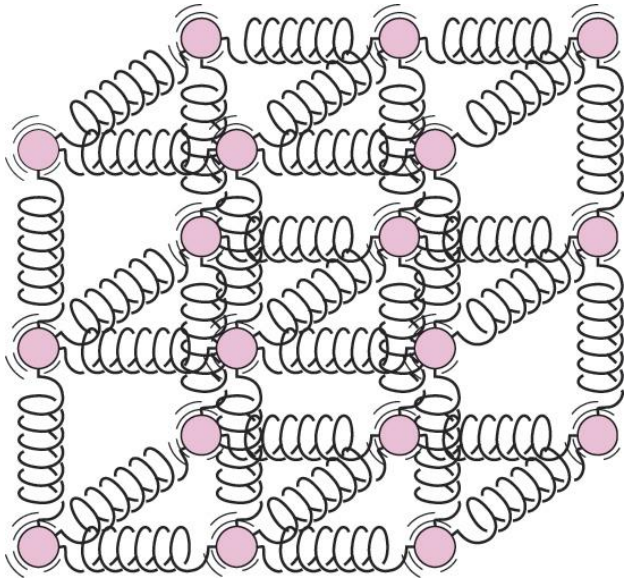
→ **mixture of oil and water is not a pure substance.**

→ **a mixture of two or more phases of a pure substance is still a pure substance.**

Phase of a Pure Substance

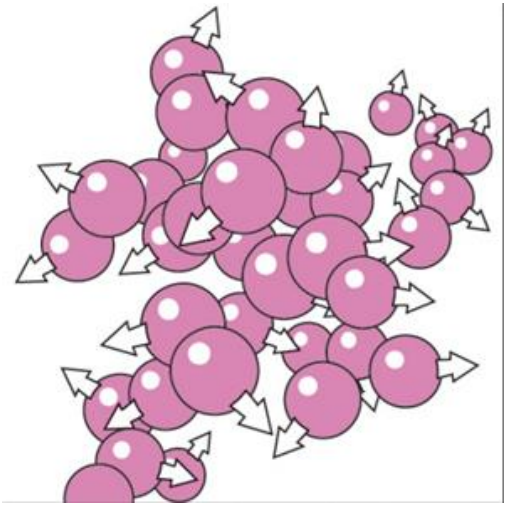
The phase of a substance is the homogeneous, chemical, and physical of aggregation of its molecules.

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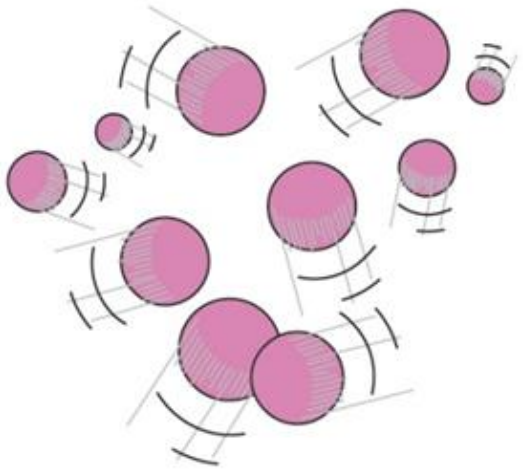
The molecules in a <u>solid</u> are kept at their positions by the large spring like inter-molecular forces.

At high temperature, molecules overcome the inter molecules forces and break away.



In the <u>liquid</u> phase the molecules are no longer at fixed positions, and chunks of the molecules float about each other.

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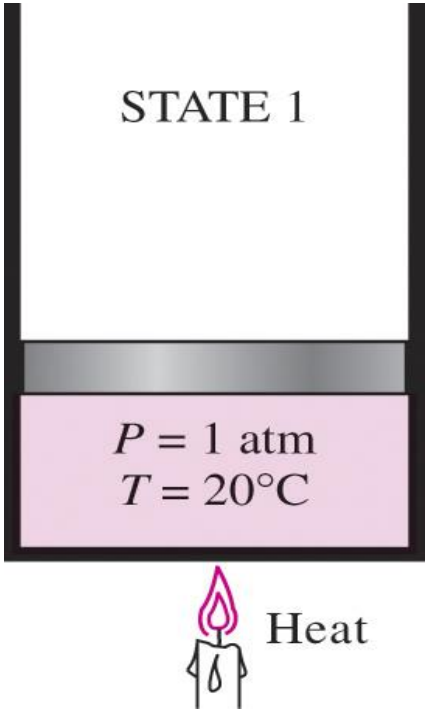


In the gas phase the molecules are far apart from each other, irregular and move about at random colliding with each other. Molecules are higher energy level than they are in liquid or solid phases.

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Phase Change of Pure Substances

Attention will be focused on liquid and vapor phases in this section. All substances exhibit general behavior. Water will be used in the following example.

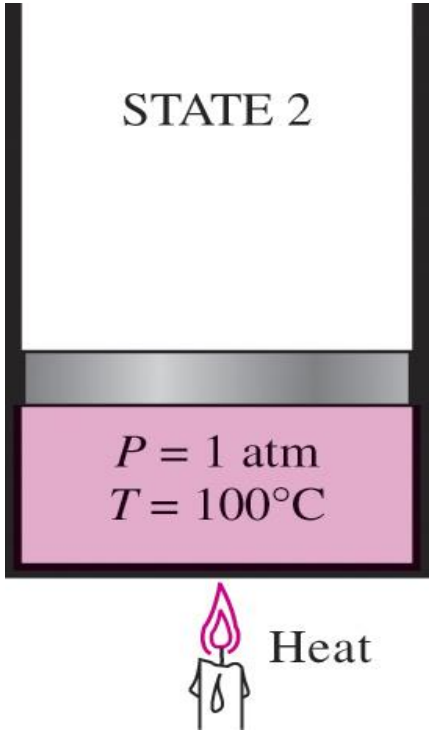


compressed liquid or subcooled liquid

At 1 atm and 20°C, water exists in the liquid phase(i.e. not about to vaporize).

As heat added the temperature increases and water expands.(i.e. v increases)

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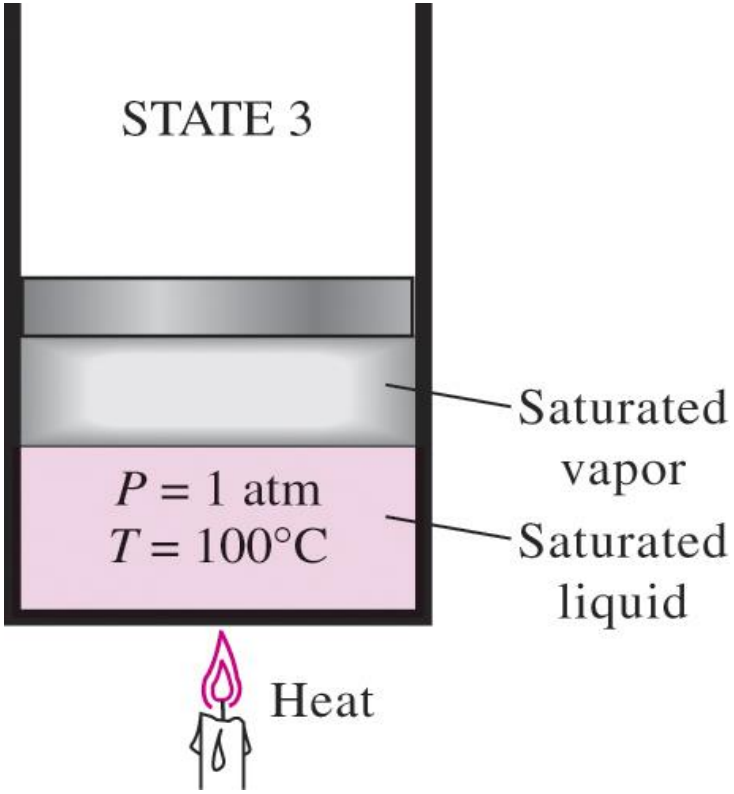


Saturated liquid

At 1 atm pressure and 100°C, water exists as a liquid that is ready to vaporize.

Any addition of heat will cause the phase change.

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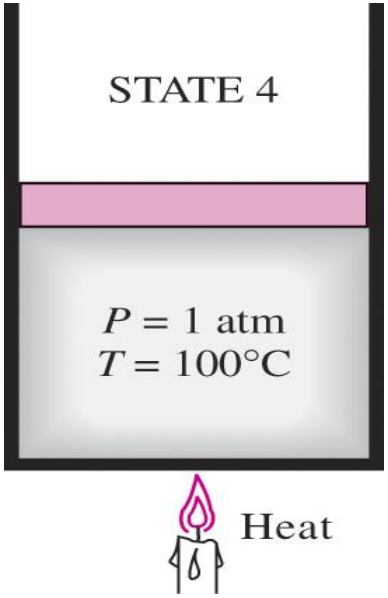


Saturated liquid–vapor mixture

The state at which the liquid and vapor phases coexist in equilibrium.

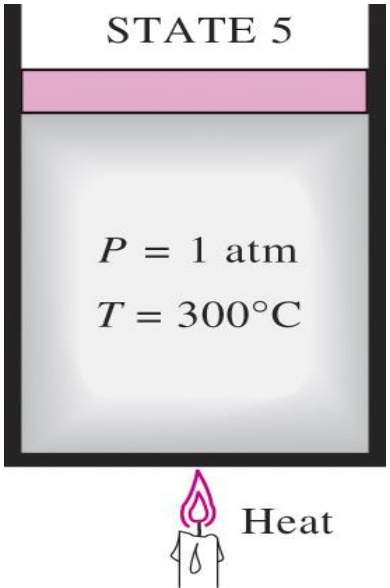
Once boiling starts, the temperature will not rise until the liquid completely vaporizes.

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Saturated vapor

A vapor that is *about to condense.*



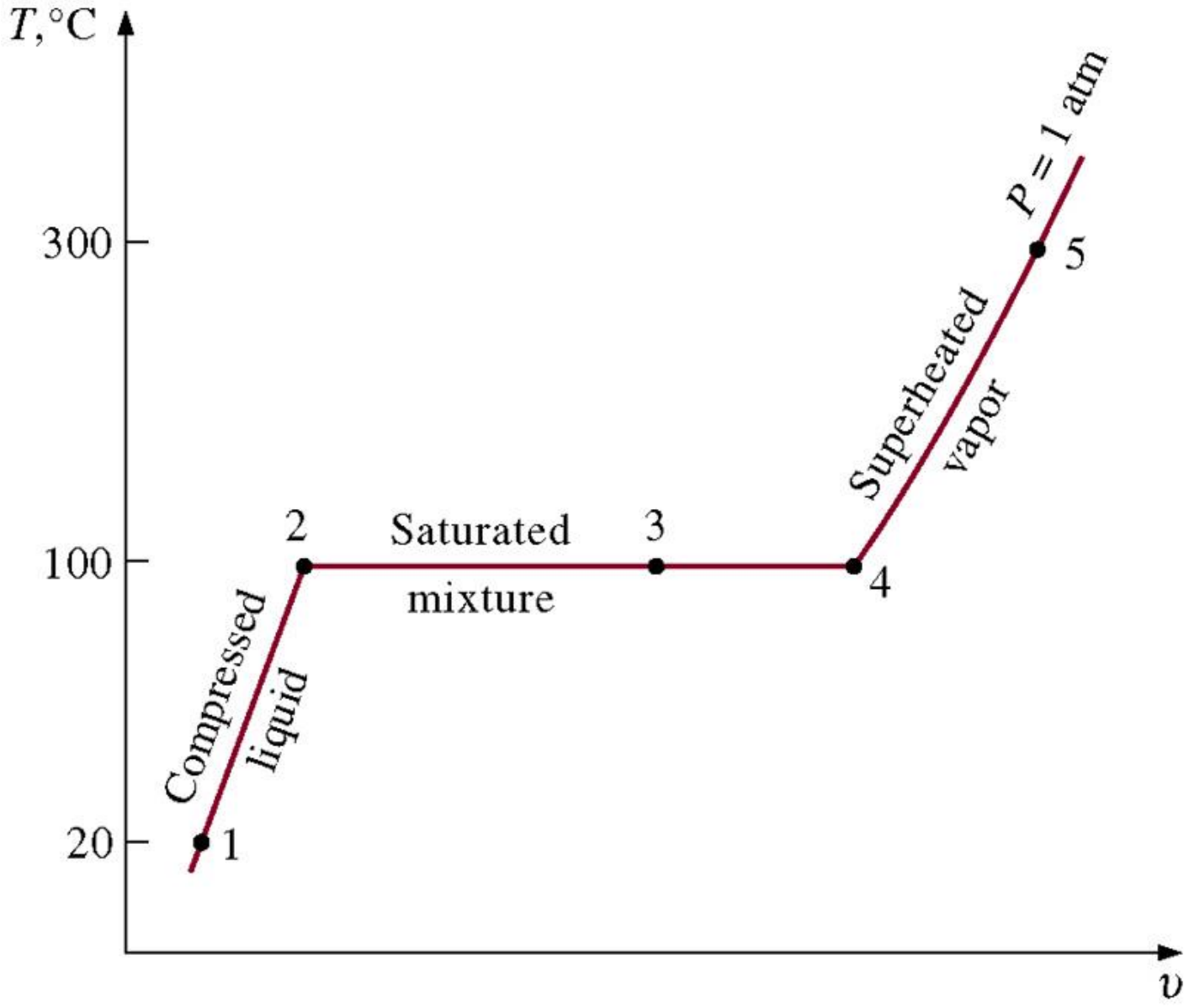
Superheated vapor

A vapor that is *not about to condense* (i.e., not a saturated vapor).

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FIGURE 2-1

***T-v* diagram for the heating process of water at constant pressure.**



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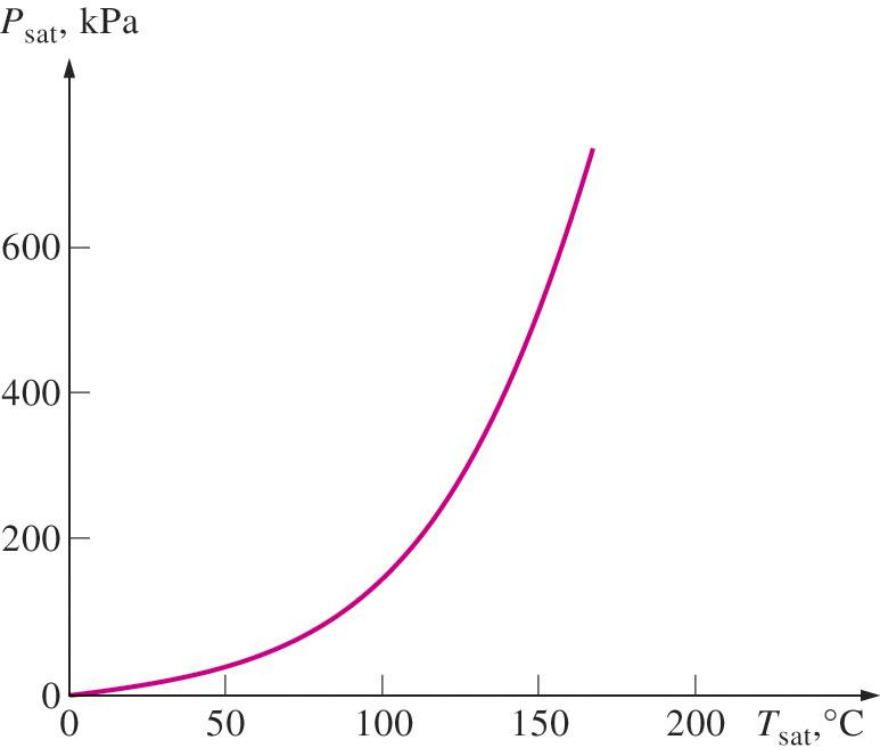


FIGURE 2-2 Liquid-vapor saturation curve

Holding the pressure constanta t 1 atm, boiling takes place at 100°C. By changing the pressure we can change the boiling temperature of water.

Saturation temperature T_{sat}

The temperature at which a pure substance starts boiling.

Saturation pressure P_{sat}

The pressure at which a pure substance starts boiling.

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Property Diagrams for phase-Change Processes

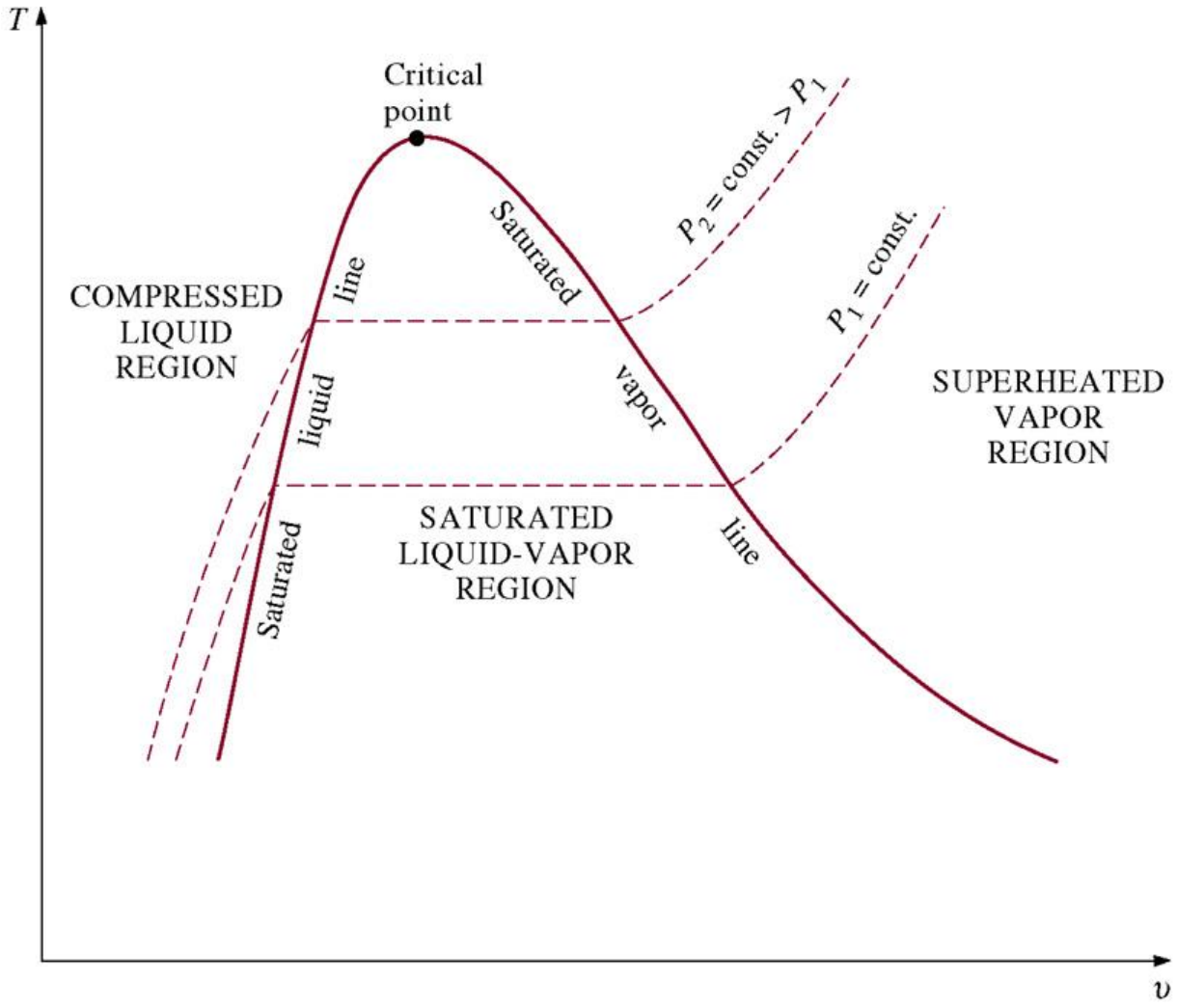


FIGURE 2-3 T-V Diagram of a Pure Substance

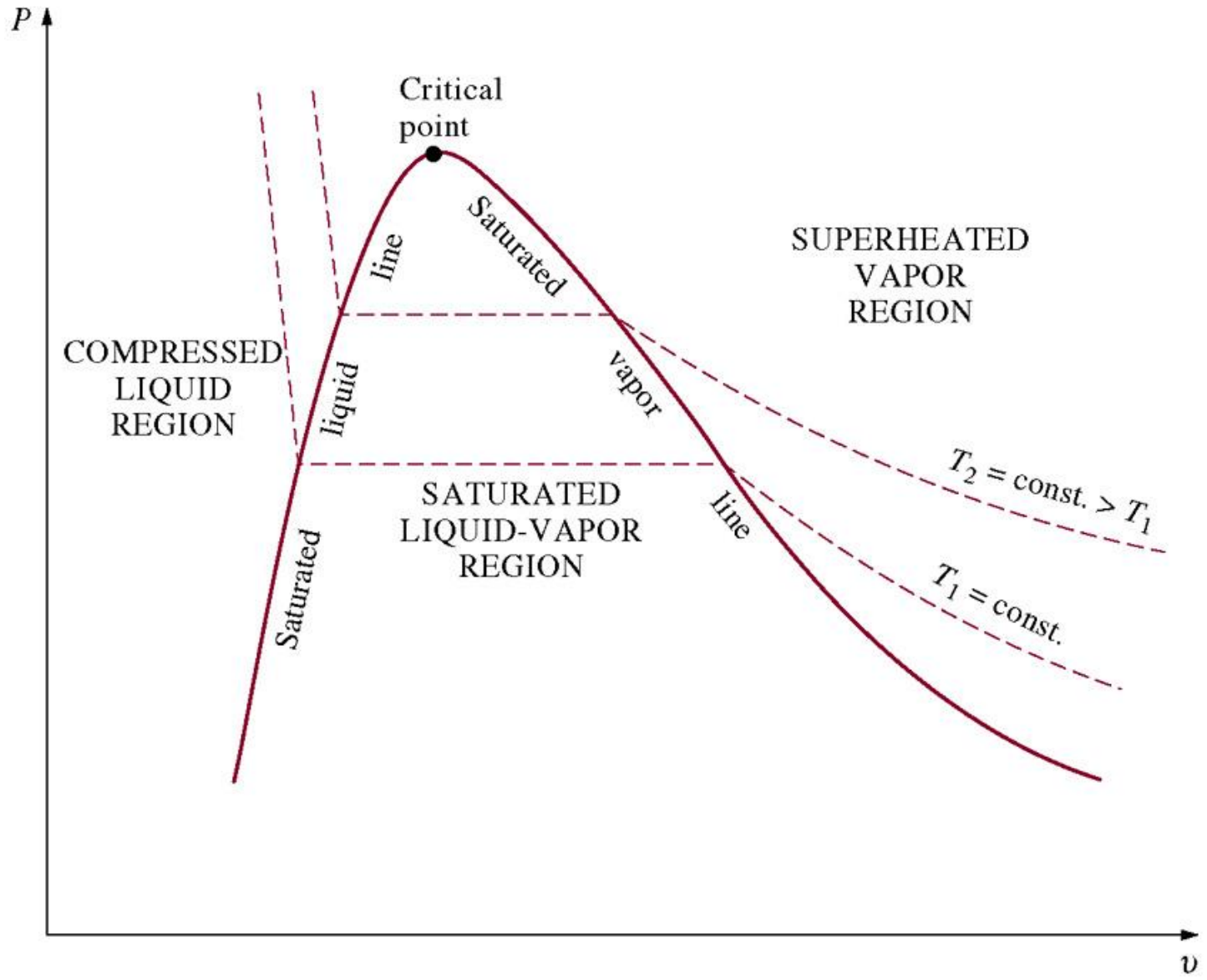
Critical point

The point at which the saturated liquid and saturated vapor states are identical.

T_c is the maximum temperature at which liquid and vapor phases can co-exist in equilibrium.

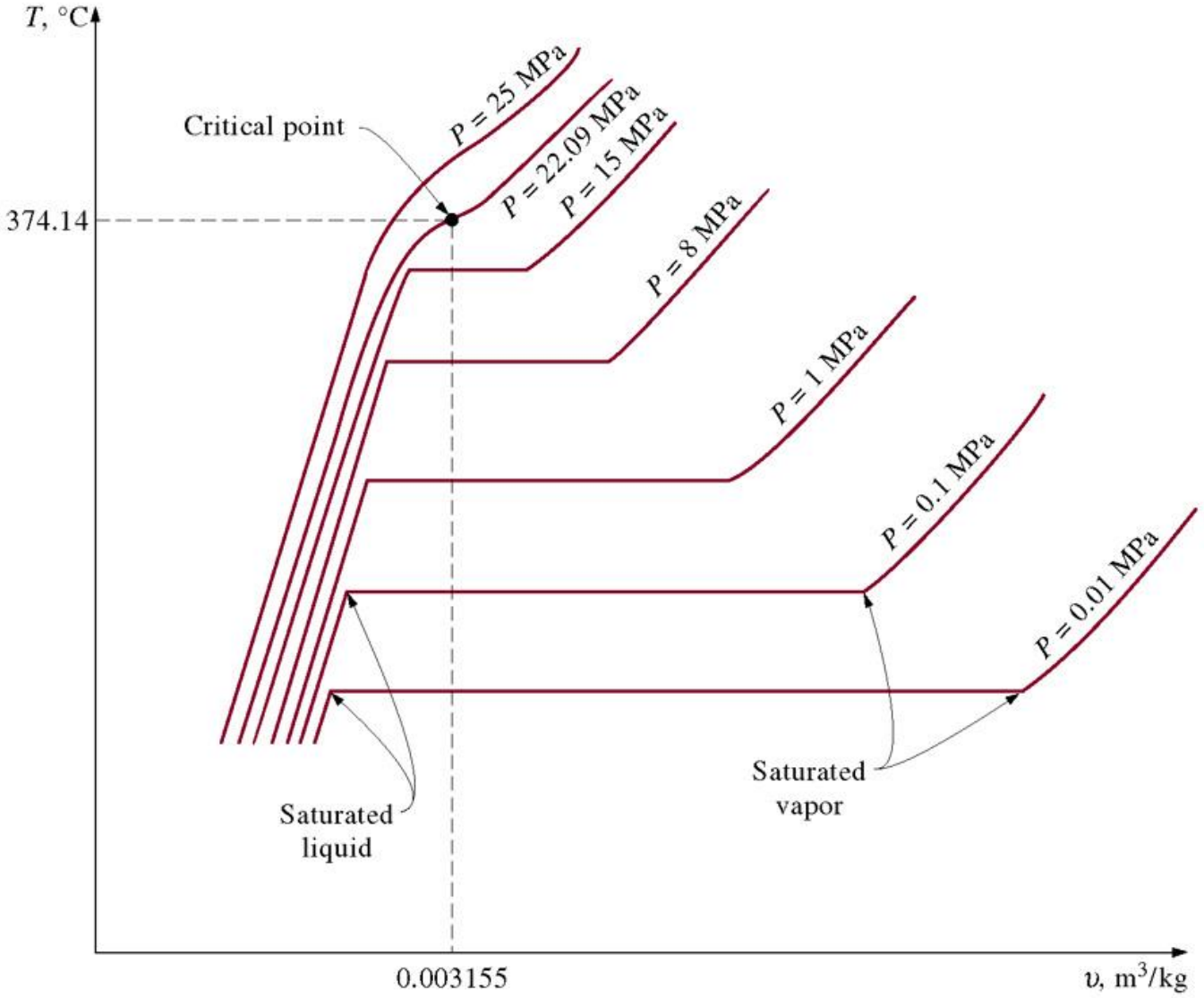
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***P-v* diagram of a pure substance.**



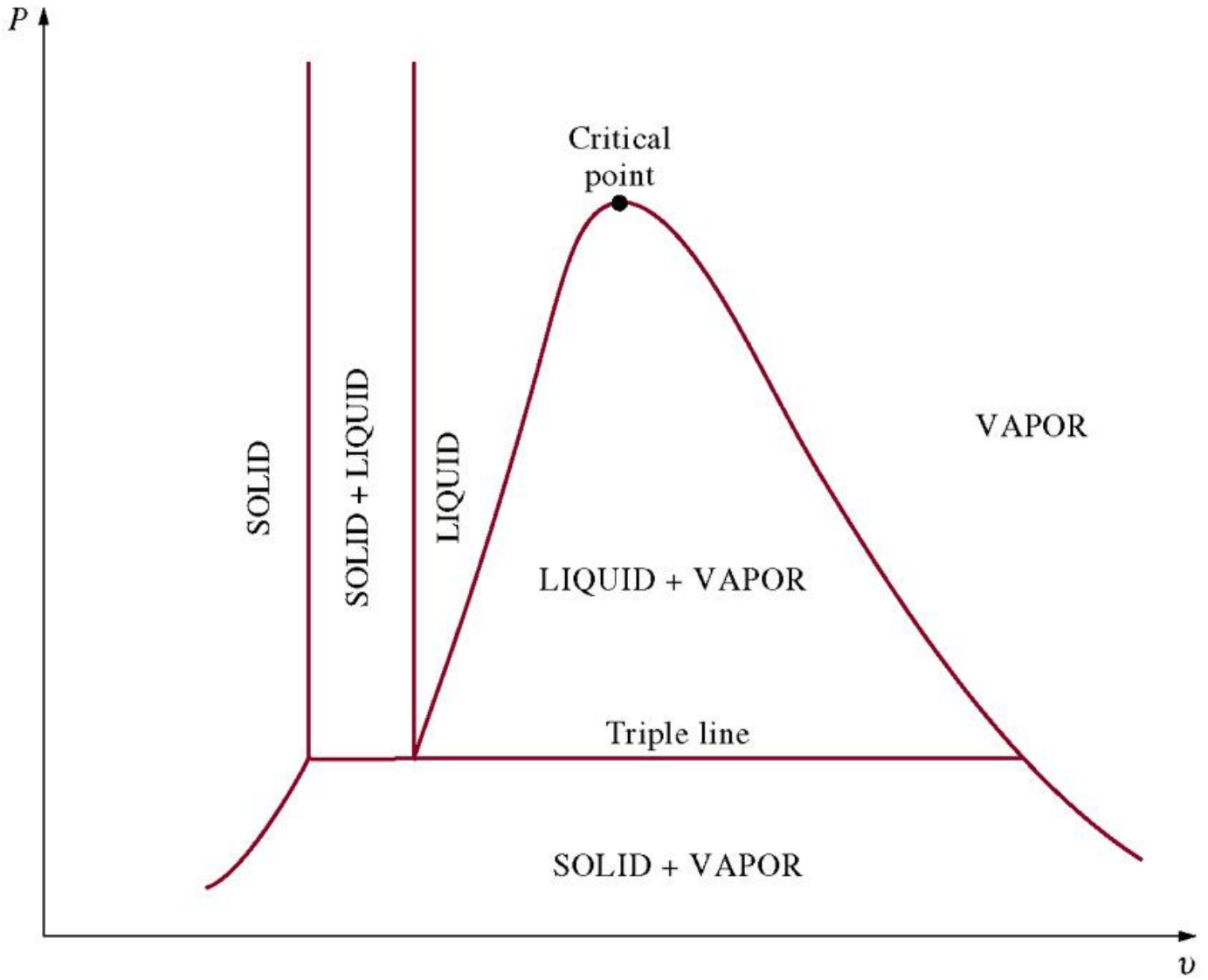
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***T-v* diagram of constant-pressure phase-change processes of a pure substance at various pressures (numerical values are for water).**



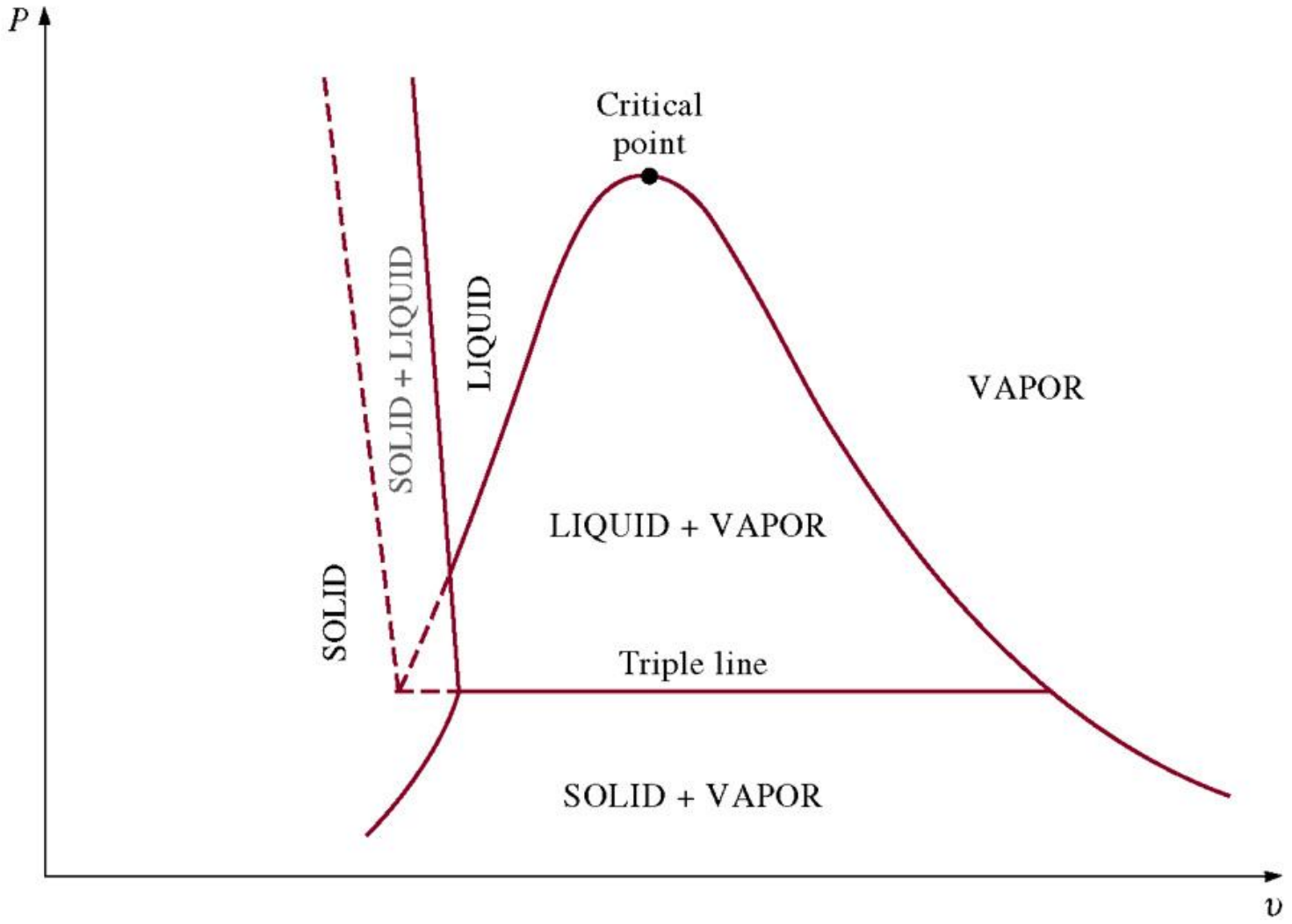
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***P-v* diagram of a substance that contracts on freezing.**



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***P-v* diagram of a substance that expands on freezing (such as water).**

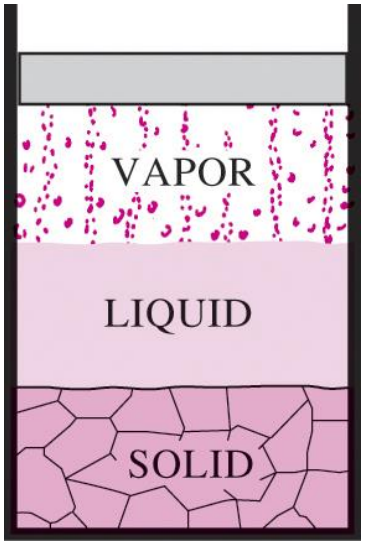


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When all three phases of a substance co-exist in equilibrium under some conditions, it is called triple phase.

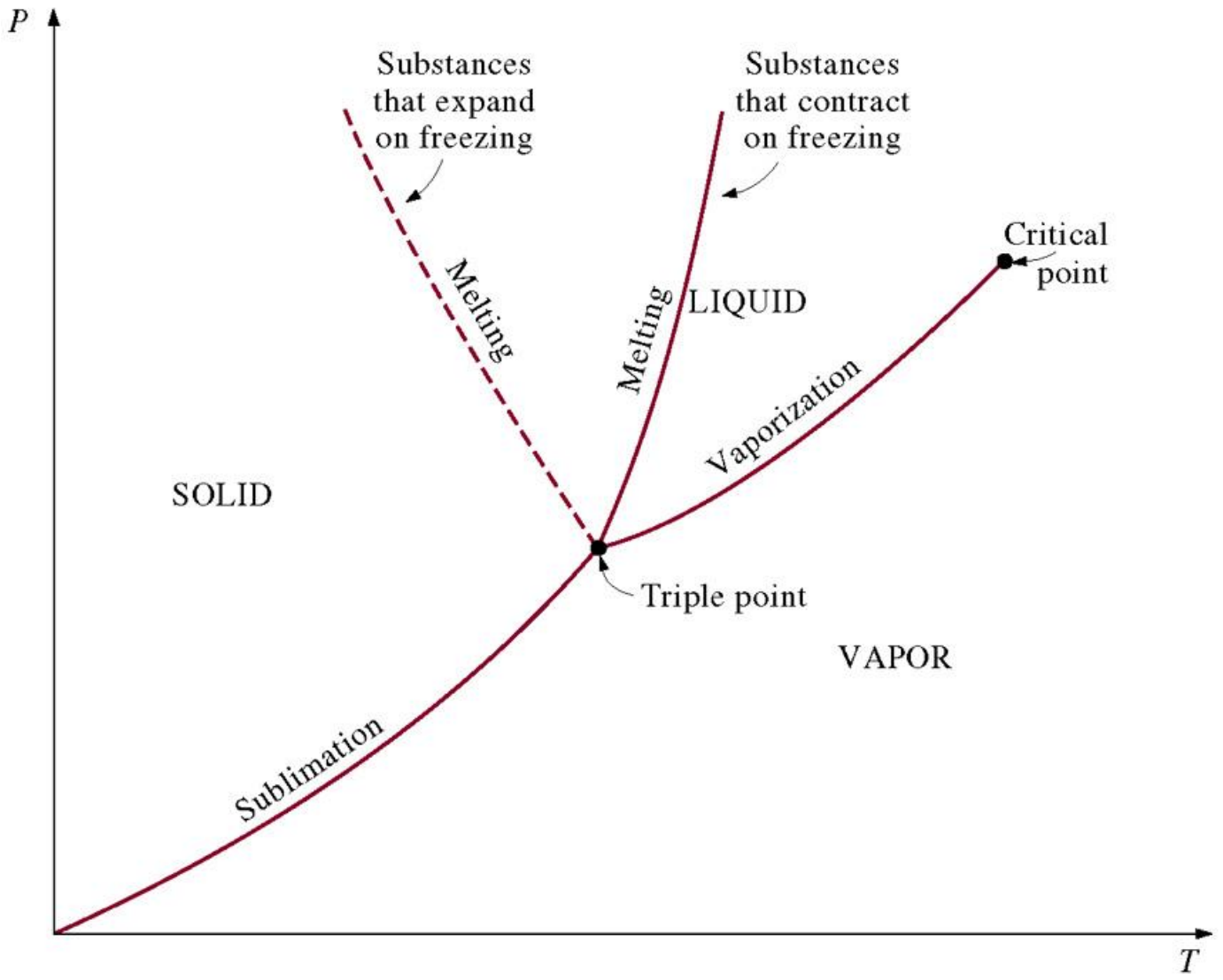
On P-v or T-v diagrams ⟶ Triple line

On P-T or T-v diagrams ⟶ Triple point



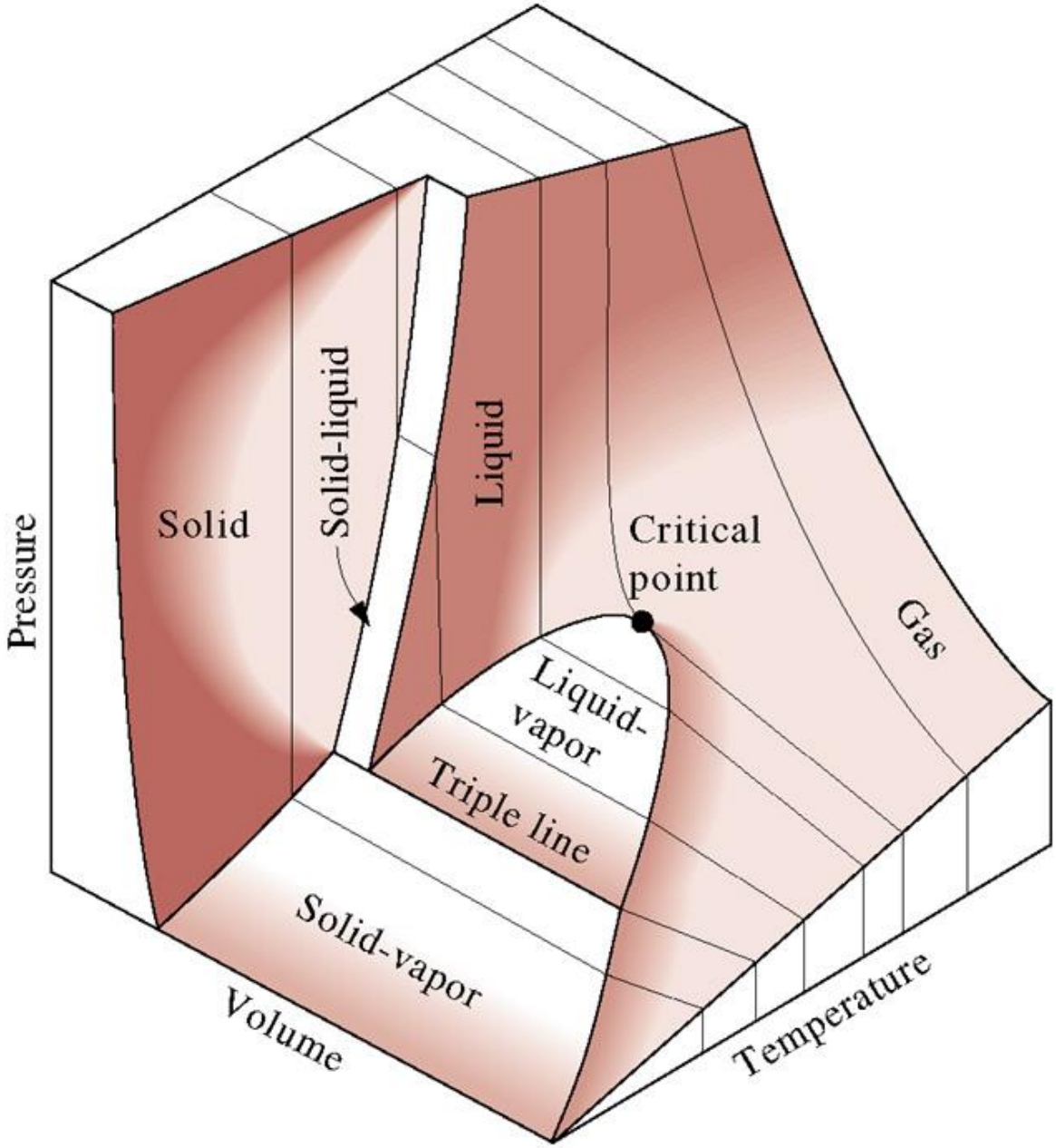
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_P-T_ diagram of pure substances.



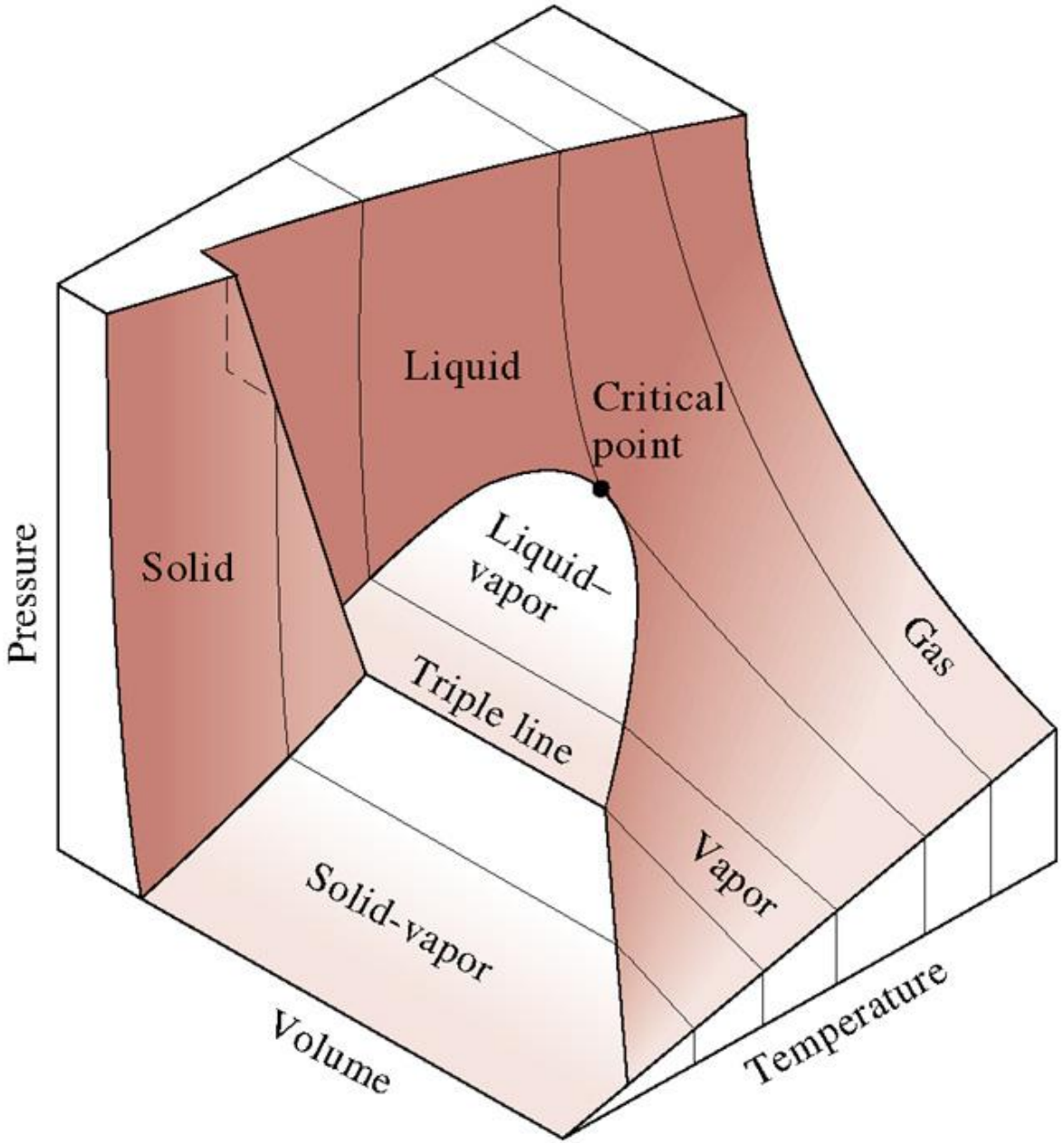
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***P-v-T* surface of a substance that *contracts* on freezing.**



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***P-v-T* surface of a substance that *expands* on freezing (like water).**



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PROPERTY TABLES

Enthalpy $\longrightarrow H = U + PV \quad (\text{kJ})$ **or** $h = u + Pv \quad (\text{kJ/kg})$

If u is not listed $\rightarrow u = h - Pv$

Saturated Liquid and Saturated Vapor States

(Tables A4 and A5 in Çengel)

Subscripts

***f* – saturated liquid**

***g* – saturated vapor**

***fg* – the difference between saturated vapor and saturated liquid**

For example:

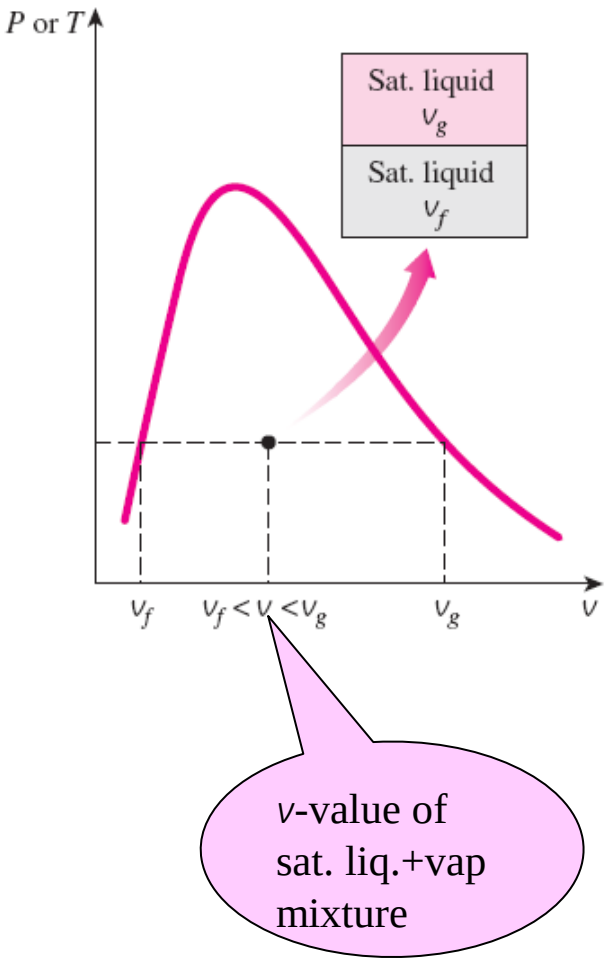
v_f = specific volume of saturated liquid

v_g = specific volume of saturated vapor

$$v_{fg} = v_g - v_f$$

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A partial list of Table A–4.



Temp. °C T	Sat. press. kPa P_{sat}	Specific volume m^3/kg Sat. liquid v_f	Sat. vapor v_g
85	57.868	0.001032	2.8261
90	70.183	0.001036	2.3593
95	84.609	0.001040	1.9808

Specific temperature

Corresponding saturation pressure

Specific volume of saturated liquid

Specific volume of saturated vapor

A partial list of Table A–5.

		.....	Enthalpy kJ/kg		
Press. P kPa	Sat temp. T_{sat} °C		Sat. liquid h_f	Evap. h_{fg}	Sat. Vapor h_g
20	60.06		251.42	2357.5	2608.9
25	64.94		271.96	2345.5	2617.5
30	69.09		289.27	2335.3	2624.6
40	75.86		317.62	2318.4	2636.1
50	81.32		340.54	2304.7	2645.2
75	91.76		384.44	2278.0	2662.4

Enthalpy of vaporization, h_{fg} (Latent heat of vaporization): The amount of energy needed to vaporize a unit mass of saturated liquid at a given temperature or pressure.

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Saturated Liquid–Vapor Mixture

In order to know the properties of the liquid and vapor phase in the mixture a new property is defined:

Quality, x : The ratio of the mass of vapor to the total mass of the mixture.
Quality is between 0 and 1 0: sat. liquid, 1: sat. vapor.

$$x = \frac{m_{\text{vapor}}}{m_{\text{total}}} \qquad m_{\text{total}} = m_{\text{liquid}} + m_{\text{vapor}} = m_f + m_g$$

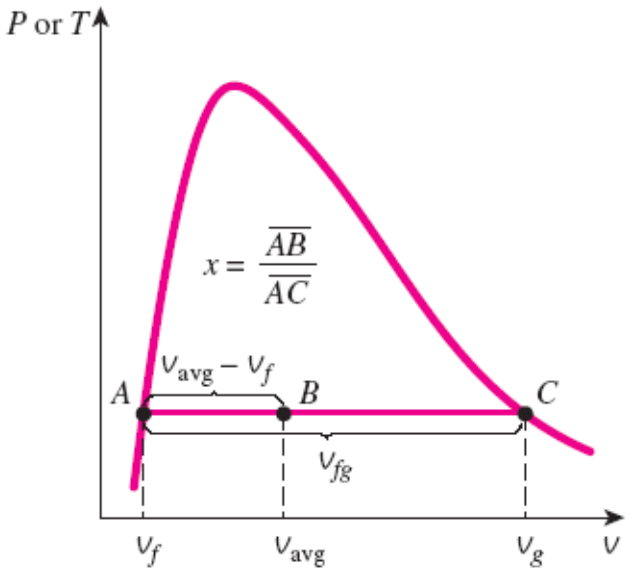
$$V = V_f + V_g$$

$$V = mv \rightarrow m_t v_{av} = m_f v_f + m_g v_g$$

$$m_f = m_t - m_g \rightarrow m_t v_{av} = (m_t - m_g) v_f + m_g v_g$$

$$\div m_t \rightarrow v_{av} = (1 - x) v_f + x v_g$$

or $v_{av} = v_f + x v_{fg}$ $\qquad x = \dfrac{v_{av} - v_f}{v_{fg}}$



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Similar equations can be derived for u or h:

$$u_{\text{avg}} = u_f + xu_{fg} \quad (\text{kJ/kg})$$

$$h_{\text{avg}} = h_f + xh_{fg} \quad (\text{kJ/kg})$$

Example **5 kg of steam at 200kPa occupied a volume of 2.60 m^3 . Determine temperature and quality.**

$$v_{av} \rightarrow v = \frac{volume}{mass} = \frac{2.60}{5} = 0.52\ m^3/kg$$

Compare this with the values given in Table A-5

Note : T_{sat}=120.23 ºC at 200 kPa (=0.200 Mpa)

v_f =0.001061 and v_g =0.8857

$$x = \frac{v_{av} - v_f}{v_{fg}} = \frac{v - v_f}{v_g - v_f} = \frac{0.52 - 0.001061}{0.8857 - 0.001061} = 0.587$$

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A partial listing of Table A–6.

	v	u	h
T, °C	m^3/kg	kJ/kg	kJ/kg
	$P = 0.1$ MPa (99.61°C)		
Sat.	1.6941	2505.6	2675.0
100	1.6959	2506.2	2675.8
150	1.9367	2582.9	2776.6
⋮	⋮	⋮	⋮
1300	7.2605	4687.2	5413.3
	$P = 0.5$ MPa (151.83°C)		
Sat.	0.37483	2560.7	2748.1
200	0.42503	2643.3	2855.8
250	0.47443	2723.8	2961.0

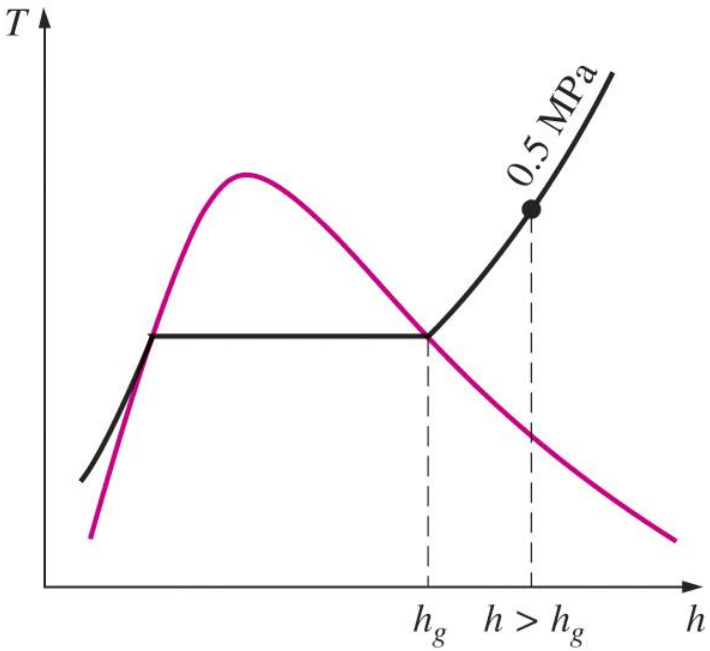
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Superheated Vapor (Table A-6)

characterized by

Lower pressures ($P < P_{sat}$ at a given T)
Higher tempreatures ($T > T_{sat}$ at a given P)
Higher specific volumes ($v > v_g$ at a given P or T)
Higher internal energies ($u > u_g$ at a given P or T)
Higher enthalpies ($h > h_g$ at a given P or T)

Example



$$at\ 0.5\ MPa \rightarrow h_g = 2748.7\ {}^{kJ}/_{kg}$$

$$T_{sat} = 151.86\ ^{\circ}C$$

However at the same pressure,
if T= 200 °C → h= 2855.4 ${}^{kJ}/_{kg}$

$$\rightarrow h > h_g$$

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Compressed Liquid

characterized by

Higher pressures ($P > P_{sat}$ at a given T)
Lower tempreatures ($T < T_{sat}$ at a given P)
Lower specific volumes ($v < v_f$ at a given P or T)
Lower internal energies ($u < u_f$ at a given P or T)
Lower enthalpies ($h < h_f$ at a given P or T)

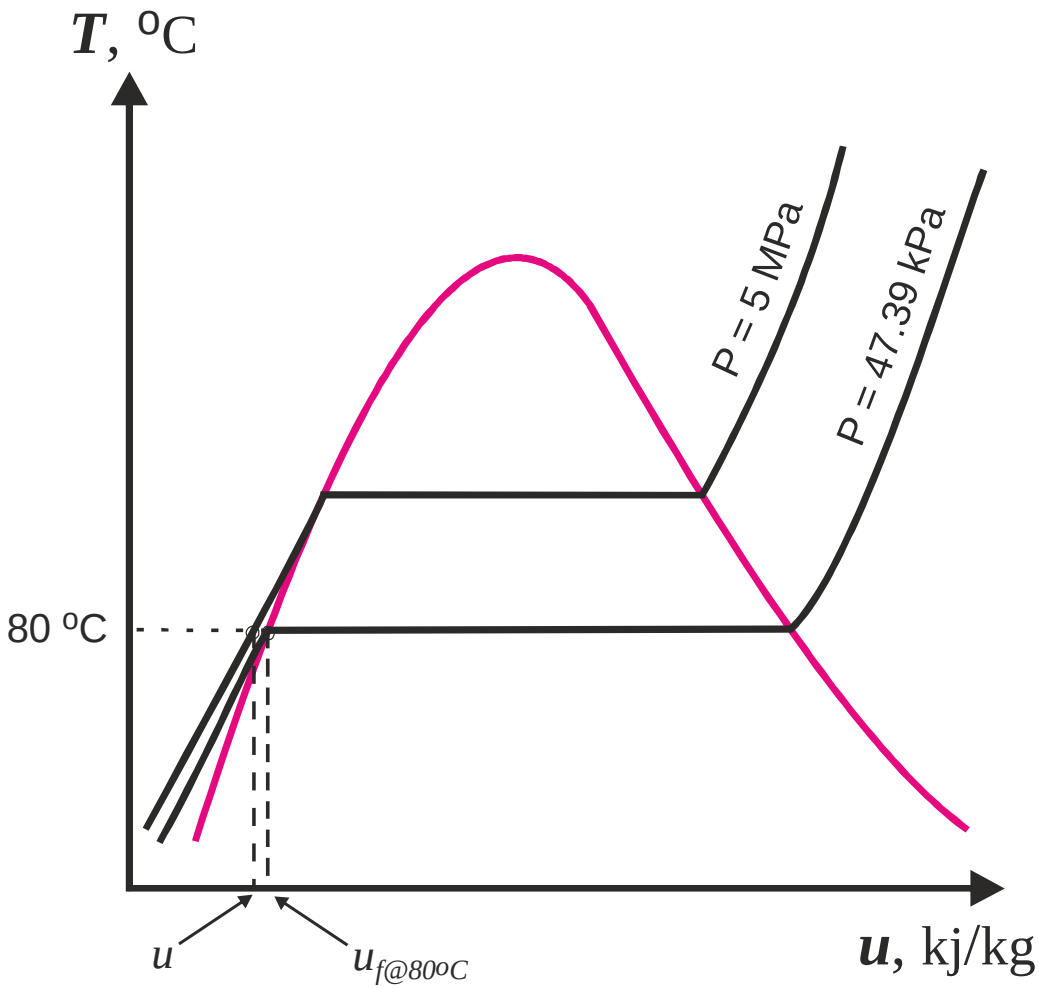
In the absence of data a compressed liquid is approximated as a saturated liquid at the given temperature.

Given: P and T

$$v \cong v_{f\,@T}$$
$$u \cong u_{f\,@T}$$
$$h \cong h_{f\,@T}$$

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Example



at 80 °C $P_{sat} = 47.39\ kPa$

If our pressure is 5 MPa then we have compressed liquid.
For compressed liquid table(A-7):

$$\left.\begin{array}{l} P = 5\ MPa \\ T = 80°C \end{array}\right\} \quad u = 333.72\ {}^{kJ}/_{kg}$$

From the saturation table(A-4):

$$u_f = 334.86\ {}^{kJ}/_{kg}$$

If we make $u \cong u_{f@80°C}$,the error is:

$$\frac{334.86 - 333.72}{333.72} = 0.0034(0.34\%)$$

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Example

State	1	2	3	4	5
P,kPa	200	300	2000	476.1	5000
T, °C	120.23	133.56	300	150	100
x, %	80	*I*	*M*	92	*M*
v, $^{m^3}/_{kg}$	0.709	*I*	0.125	0.361	0.00104

$$state\ 1 (table\ A-5) \rightarrow T = T_{sat@200kPa} = 120.23°C$$

$$v = v_f + x v_{fg}$$

$$= 0.001061 + 0.8(0.8857 - 0.001061) = 0.709$$

$$state\ 2 \rightarrow at\ 300\ kPa \quad T_{sat} = 133.55°C$$

x = indeterminate = *I*

v = indeterminate = *I*

either saturated liquid
or saturated vapor
or saturated liquid-vapor mixture

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$state\ 3 \rightarrow at\ 2000\ kPa\ \ T_{sat} = 212.42°C$

$\therefore Superheated(\ Table\ A-6)$

x=meaning less=M

v=012547 $^{m^3}/_{kg}$

$state\ 4 \rightarrow at\ 150°C\ \ P_{sat} = 0.4758MPa\ \ v_f = 0.001091 \qquad v_g = 0.3928\ ^{m^3}/_{kg}$

$$x = \frac{v - v_f}{v_g - v_f} = \frac{0.361 - 0.001091}{0.3928 - 0.001091} = 0.919(or\ 92\%)$$

$state\ 5 \rightarrow at\ 5000kPa\ \ T_{sat} = 263.99°C$ **Compressed liquid** → x=meaningless=M

$Table\ (A-7)\ at\ 5MPa\ and\ 100°C \qquad v = 0.001041\ ^{m^3}/_{kg}$

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The Ideal Gas Equation of State

Equation of State: any equation that relates the pressure, temperature, and specific of a substance.

Specific volume

Ideal-gas equation of state $\longrightarrow$ $P = R\left(\frac{T}{v}\right)$ or $Pv = RT$

Absolute temperature

Absolute pressure

Gas constant

Ideal gas $\longrightarrow$ **is a gas which obeys the above relation. The gas constant *R* is different for each gas.**

$$R = \frac{R_u}{M}\ {}^{kJ}/_{kg.K}$$

R_u =universal gas constant

M=molar mass(molecules weight)

R_u = 8.314 ${}^{kJ}/_{(kmol.K)}$ or 1.986 ${}^{Btu}/_{(lbmol.R)}$ (same for all substances)

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$$m = MN \qquad (\text{kg})$$

Mass of system

Mole number

Ideal Gas Equation in different forms:

$$V = mv \longrightarrow PV = mRT$$

State 1 $\rightarrow P_1V_1 = mRT_1$

State 2 $P_2V_2 = mRT_2$

$$mR = (MN)R = NR_u \longrightarrow PV = NR_uT$$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} = \text{mR}$$

$$V = N\bar{v} \longrightarrow P\bar{v} = R_uT$$

Molar specific volume

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Example

Determine the density and specific volume of air at room temperature.

Room temperature = 20°C

273+20 = 293 K

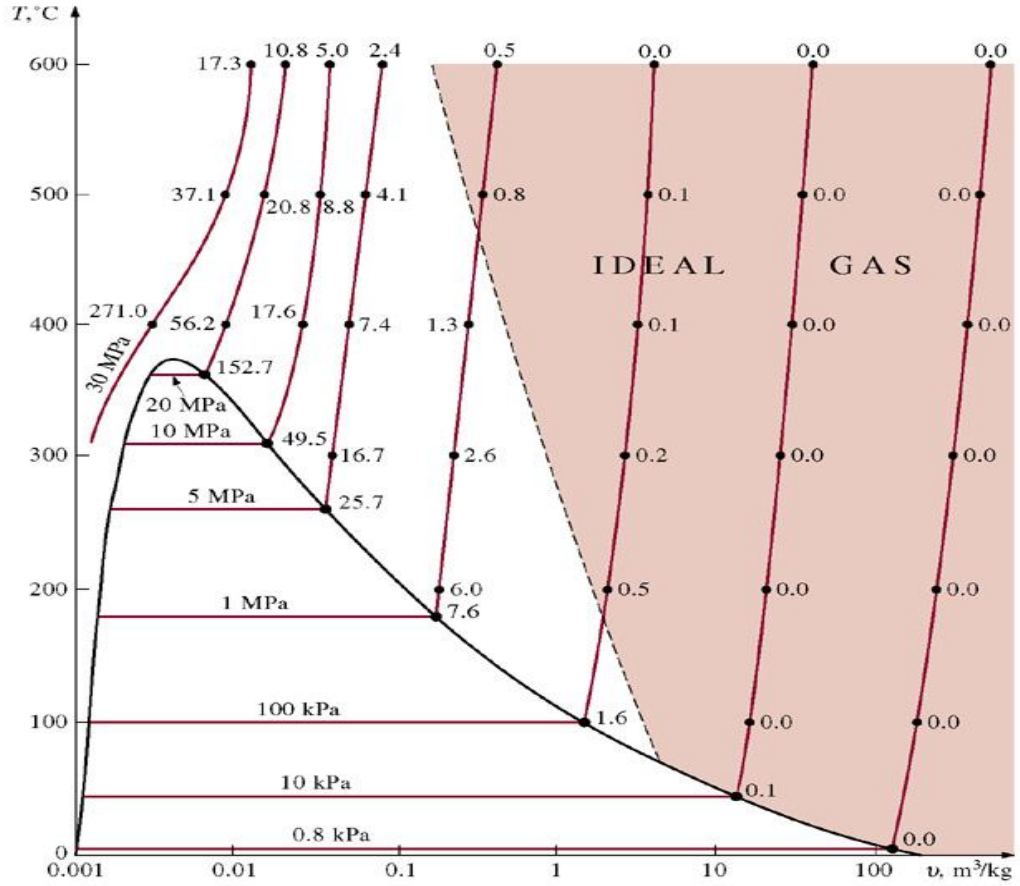
$\rightarrow PV = m\,RT \qquad P\,{}^{1}/_{\rho} = RT$

$$\rho = \frac{P}{RT} = \frac{101\,(kPa)}{0.287\left({}^{kJ}/_{kg}\right)293\,(K)} = 1.2\ {}^{kg}/_{m^3}$$

$$v = \frac{1}{\rho} = 0.83\ {}^{m^3}/_{kg}$$

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FIGURE 2-14
Percentage of error involved in assuming steam to be an ideal gas, and the region where steam can be treated as an ideal gas with less than 1 percent error.



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COMPRESSIBILITY FACTOR

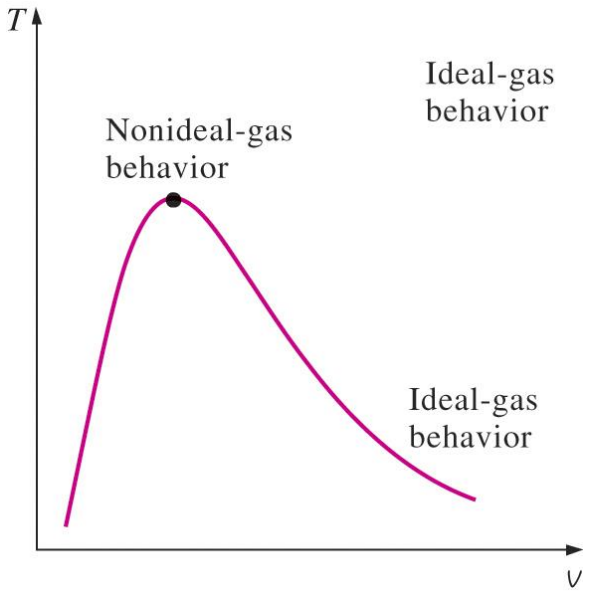
Gases deviate from ideal-gas behavior significantly at state near saturation region and the critical point.
Hence a correction is introduced:

$\longrightarrow$ **compressibility factor =** $Z = \dfrac{P\vee}{RT}$ **(for ideal gases Z=1)**

Z is an indication of deviation from ideal gas behavior.
Gases behave differently at a given temperature and pressure.

$$P_R = \frac{P}{P_{cr}} \qquad T_R = \frac{T}{T_{cr}}$$

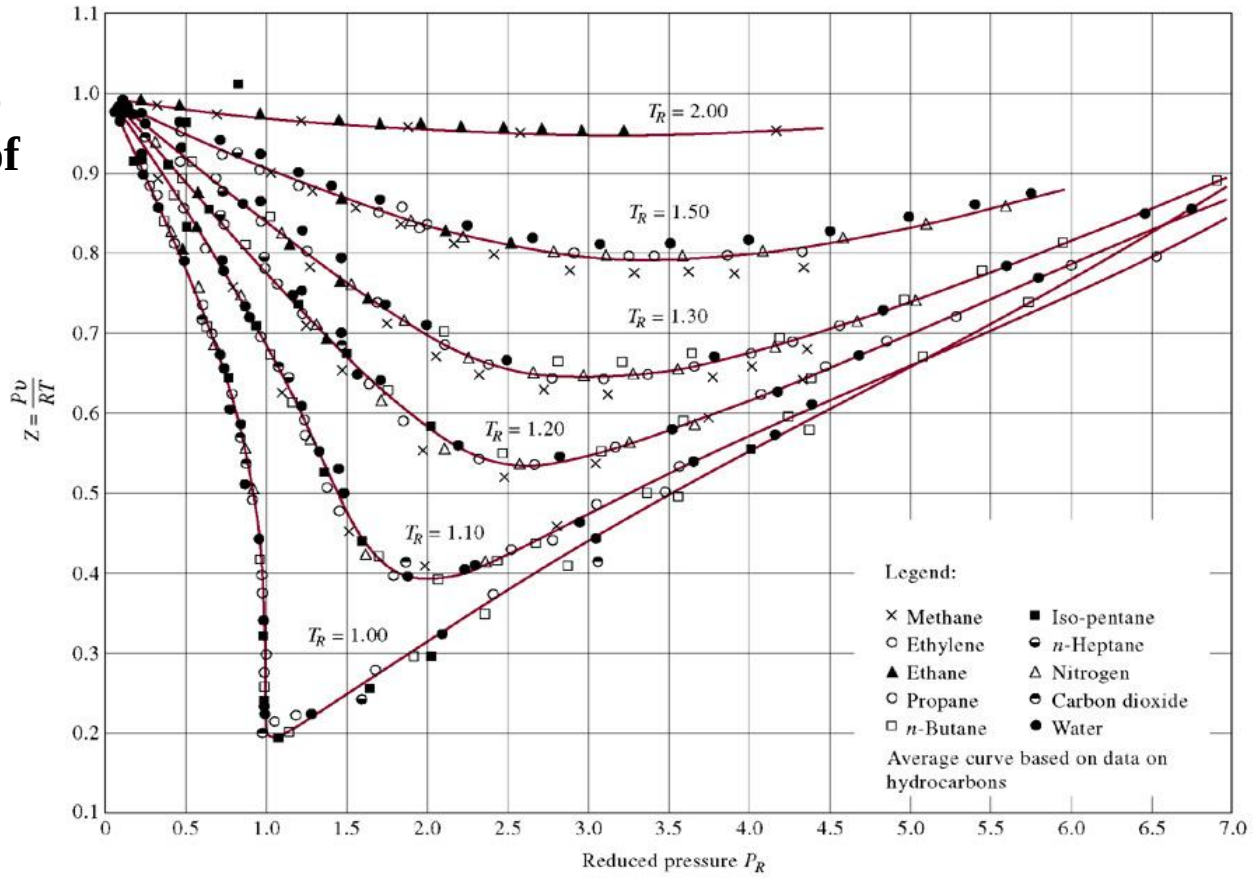
Reduced Pressure **Reduced Temperature**



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The *Z* – factor is approximately the same for all gases at the same reduced temperature and pressure.

FIGURE 2-13
Comparison of Z factors for various gases.



1) $P_R \ll 1$, gases behave as an ideal gas regardless of T.
2) $T_R > 2$, ideal gas assumed regardless of P(except $P_R \gg 1$)
3) The deviation from ideal-gas is greatest around critical point (i.e. $T_R = 1$, $P_R = 1$)

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Example

Determine the specific volume of R-134-a at 1 MPa and 50°C.

⟶ **if ideal gas equation used**

$$\rightarrow v = \frac{RT}{P} = \frac{0.08149 \times 323}{1000} = 0.02632 \ \ m^3/kg$$

Z can be determined:

$$P_R = \frac{P}{P_{cr}} = \frac{1MPa}{4.067MPa} = 0.245$$

$$T_R = \frac{T}{T_{cr}} = \frac{323K}{374.3\,K} = 0.862$$

$$Z = 0.835 \rightarrow v = Zv_{ideal}$$

$$v = 0.835 \times 0.02632$$

$$= 0.02197 \ \ m^3/kg$$

OTHER EQUATIONS OF STATE

Several equations have been proposed to represent the *P*-*v*-*T* behavior of substances accurately over a larger region with no limitations.

van der Waals
Berthelet
Redlich-Kwang
Beattie-Bridgeman
Benedict-Webb-Rubin
Strobridge
Virial

Van der Waals Equation of State

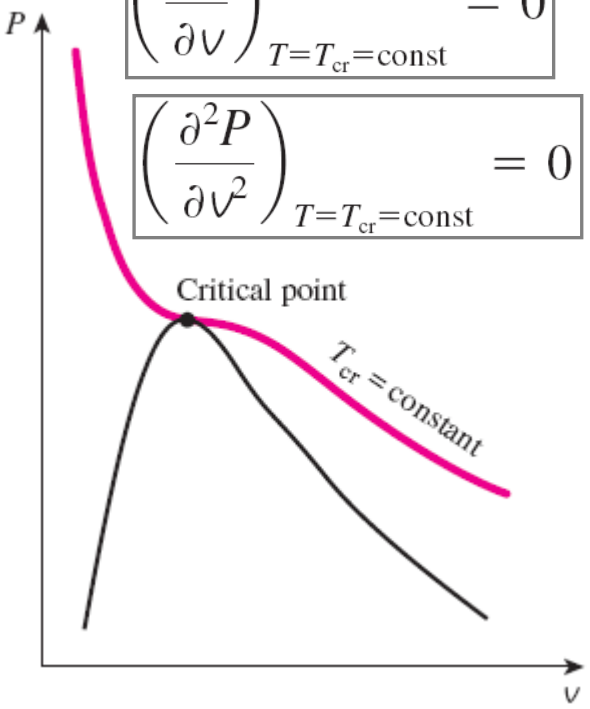
$$\left(P + \frac{a}{v^2}\right)(v - b) = RT$$

$$a = \frac{27R^2T_{cr}^2}{64P_{cr}} \qquad b = \frac{RT_{cr}}{8P_{cr}}$$

Critical isotherm of a pure substance has an inflection point at the critical state.

$$\left(\frac{\partial P}{\partial v}\right)_{T=T_{cr}=\text{const}} = 0$$

$$\left(\frac{\partial^2 P}{\partial v^2}\right)_{T=T_{cr}=\text{const}} = 0$$



This model includes two effects not considered in the ideal-gas model: the *intermolecular attraction forces* and the *volume occupied by the molecules themselves*. The accuracy of the van der Waals equation of state is often inadequate.

Beattie-Bridgeman Equation of State

$$P = \frac{R_u T}{\bar{v}^2}\left(1 - \frac{c}{\bar{v} T^3}\right)(\bar{v} + B) - \frac{A}{\bar{v}^2}$$

$$A = A_0\left(1 - \frac{a}{\bar{v}}\right) \quad B = B_0\left(1 - \frac{b}{\bar{v}}\right)$$

The constants are given in Table 3–4 for various substances. It is known to be reasonably accurate for densities up to about $0.8\rho_{cr}$.

Benedict-Webb-Rubin Equation of State

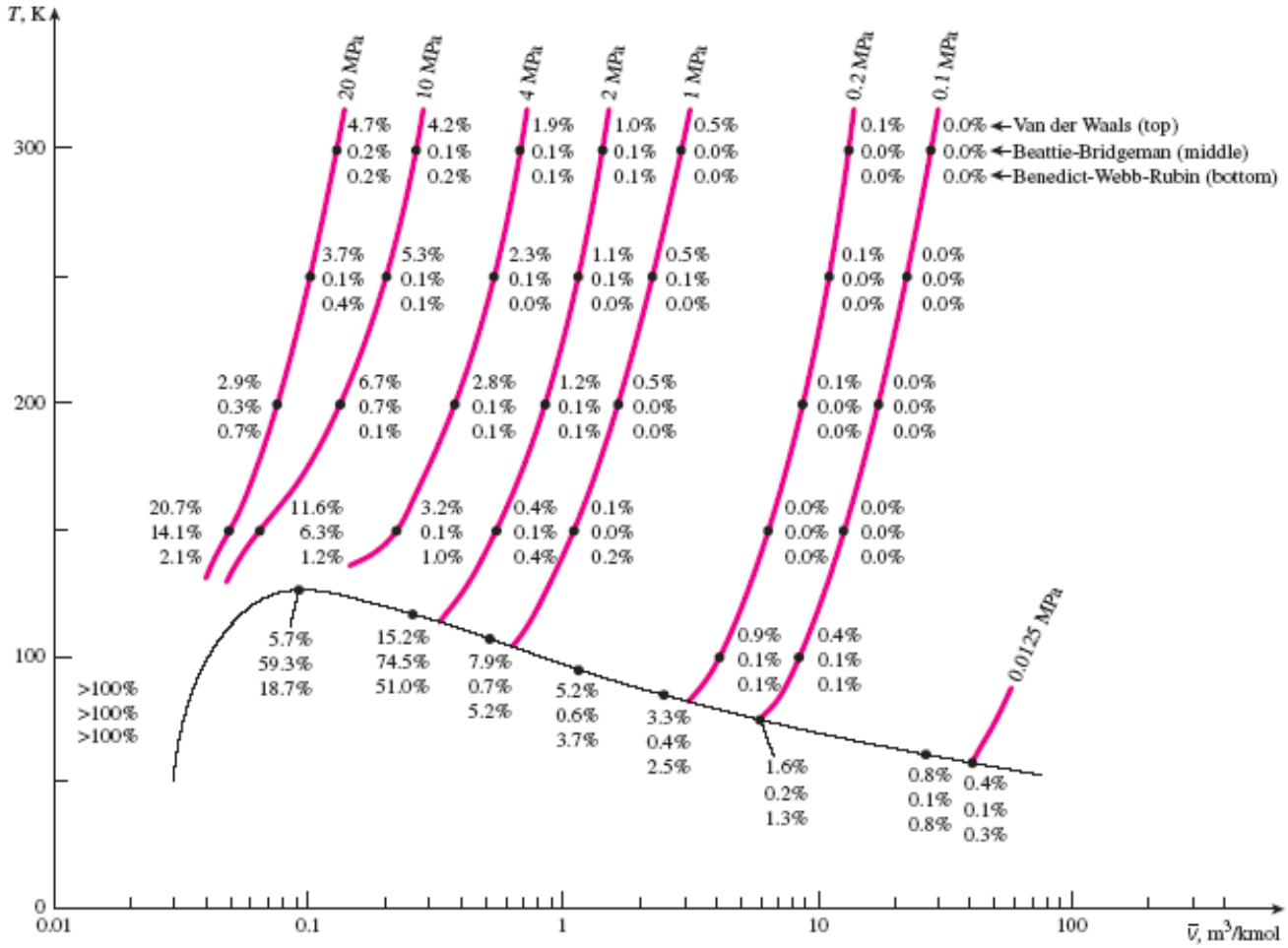
$$P = \frac{R_u T}{\bar{v}} + \left(B_0 R_u T - A_0 - \frac{C_0}{T^2}\right)\frac{1}{\bar{v}^2} + \frac{bR_u T - a}{\bar{v}^3} + \frac{a\alpha}{\bar{v}^6} + \frac{c}{\bar{v}^3 T^2}\left(1 + \frac{\gamma}{\bar{v}^2}\right)e^{-\gamma/\bar{v}^2}$$

The constants are given in Table 3–4. This equation can handle substances at densities up to about 2.5 ρ_{cr}.

Virial Equation of State

$$P = \frac{RT}{v} + \frac{a(T)}{v^2} + \frac{b(T)}{v^3} + \frac{c(T)}{v^4} + \frac{d(T)}{v^5} + \cdots$$

The coefficients $a(T)$, $b(T)$, $c(T)$, and so on, that are functions of temperature alone are called *virial coefficients*.



van der Waals: 2 constants.
Accurate over a limited range.

Beattie-Bridgeman: 5 constants.
Accurate for $\rho \leq 0.8\rho_{cr.}$

Benedict-Webb-Rubin: 8 constants.
Accurate for $\rho \leq 2.5\rho_{cr.}$

Strobridge: 16 constants.
More suitable for computer calculations.

Virial: may vary.
Accuracy depends on the number of terms used.

Complex equations of state represent the *P-v-T* behavior of gases more accurately over a wider range.

Percentage of error involved in various equations of state for nitrogen
(% error = $[(|v_{table} - v_{equation}|)/v_{table}] \times 100$).

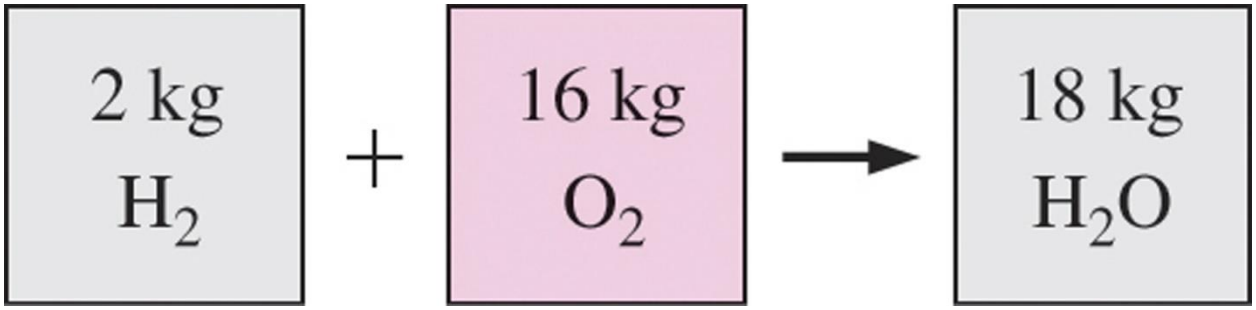
CHAPTER 2

The First Law of Thermodynamics for *Control Volumes*

Conservation of mass: Mass, like energy, is a conserved property, and it cannot be created or destroyed during a process.

Closed systems: The mass of the system remain constant during a process.

Control volumes: Mass can cross the boundaries, and so we must keep track of the amount of mass entering and leaving the control volume.



Mass is conserved even during chemical reactions.

Mass m and energy E can be converted to each other according to $E = mc^2$

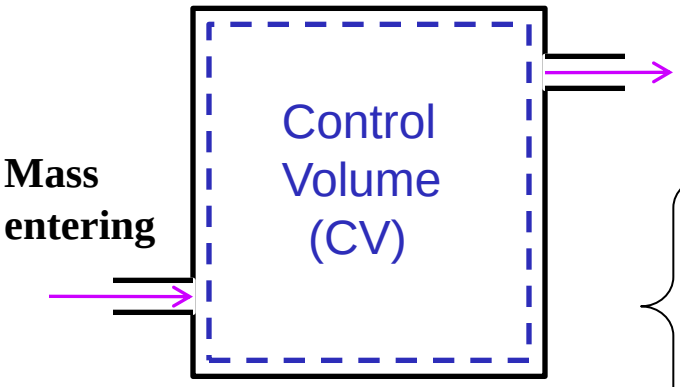
where c is the speed of light in a vacuum, which is $c = 2.9979 \times 10^8$ m/s.

The mass change due to energy change is absolutely negligible.

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Conservation of mass principle

Control volume can be thought of a region of space through which mass flows.



Total mass entering CV − **Total mass leaving CV** = **Net change in mass within CV**

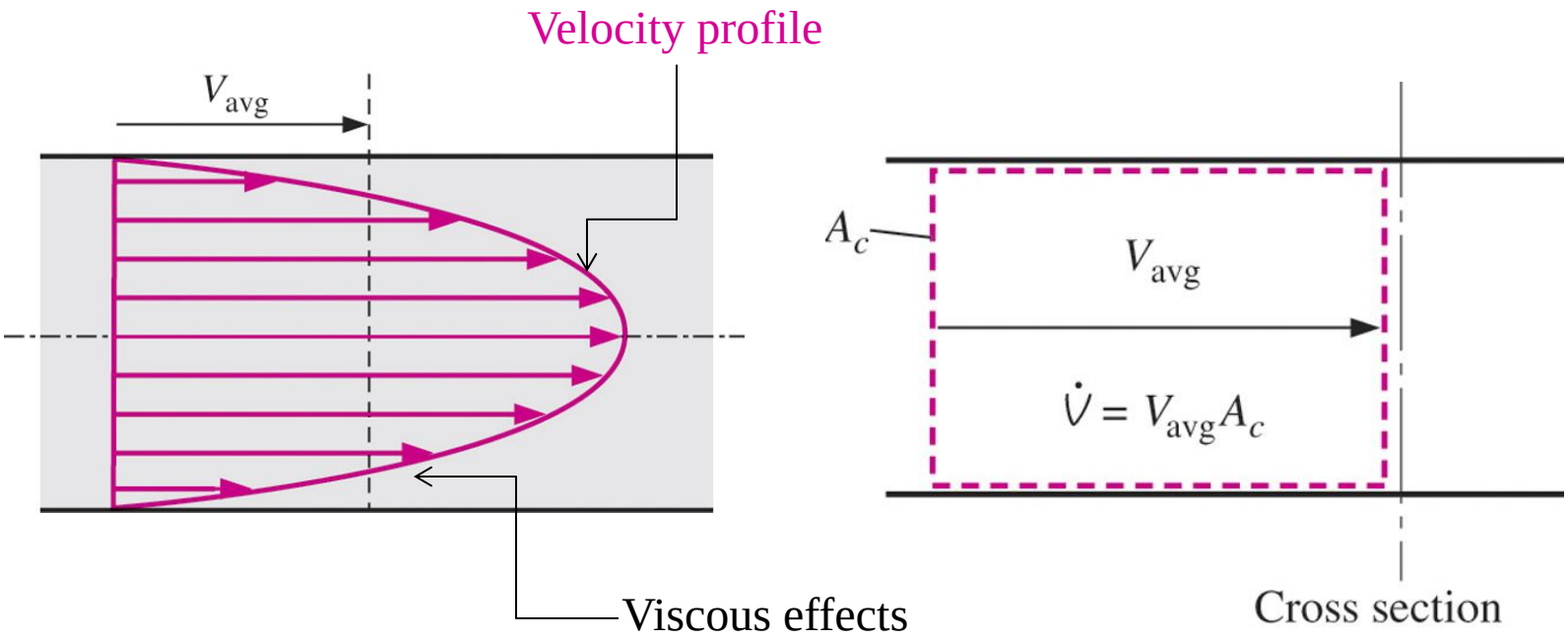
$$\sum m_i - \sum m_e = \Delta m_{\mathrm{CV}}$$

Note: *i*-inlet , *e*-exit , CV-control volume

The rate form $\rightarrow$ $$\sum \dot{m}_i - \sum \dot{m}_e = \Delta \dot{m}_{\mathrm{CV}}$$

where $\dot{m}$ **is mass flow rate, i.e** $\frac{dm}{dt}$

Flow through a pipe or duct

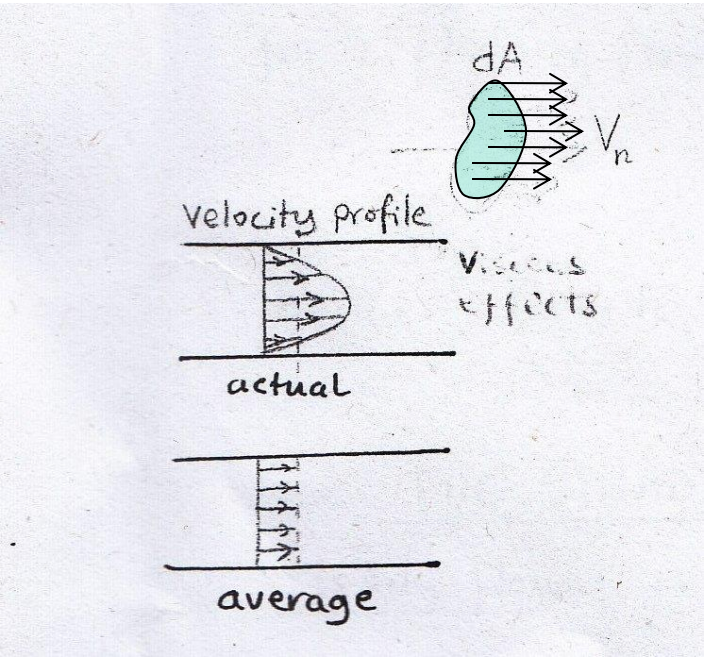


The average velocity V_{avg} is defined as the average speed through a cross section.

The volume flow rate is the volume of fluid flowing through a cross section per unit time.

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The mass flow rate through a different area *dA* can be expressed as:



$$d\dot{m} = \rho V_n dA$$

↓

Velocity normal to *dA*

$$\dot{m} = \int_A \rho V_n dA \quad (\text{kg/s})$$

$$\dot{m} = \rho V_{avg} A \quad (\text{kg/s})$$

density kg/m^3 **average fluid velocity normal to *A*,** m/s

The volume flow rate:

$$\dot{V} = \int_A V_n dA = V_{av} A \left(m^3/s\right)$$

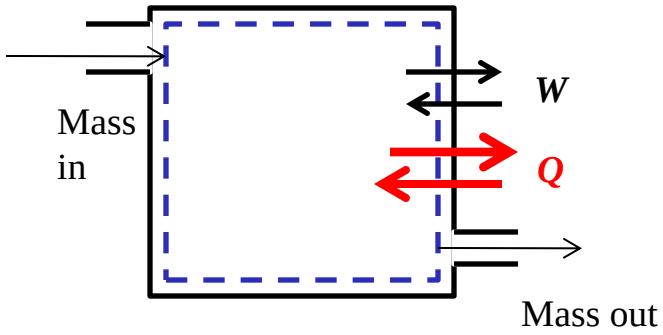
The mass and volume flow rates are related by :

$$\dot{m} = \rho \dot{V} = \frac{\dot{V}}{v}$$

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Conservation of Energy Principle

{Total energy crossing boundary as heat and work} + {Total energy of mass entering CV} - {Total energy of mass exiting CV} = {Net change in energy of CV}



$$Q - W + \sum E_{in} - \sum E_{out} = \Delta E_{CV}$$

This equation can also be expressed in the rate form (i.e. quantities per unit time).

When there is no mass flow in and out of the system, the energy equation reduces to that of a closed system:

$$Q - W = \Delta E$$

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Flow work or flow energy

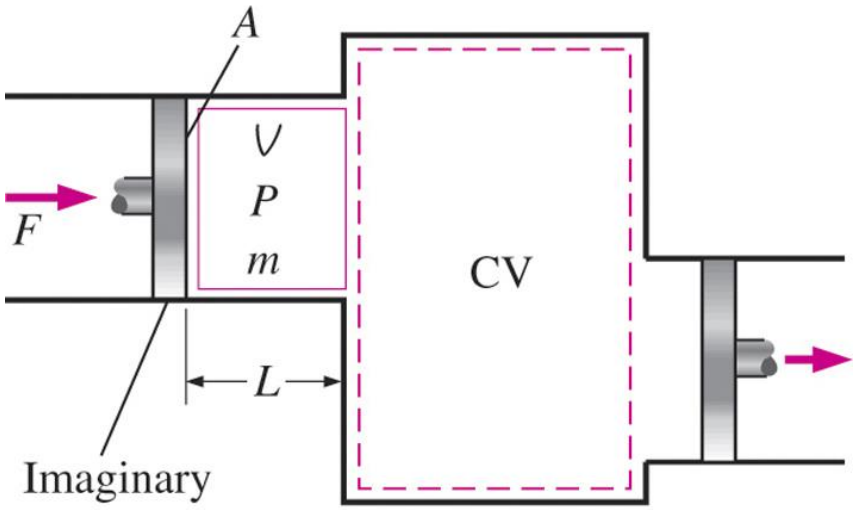
The energy required to push fluid into or out of a control volume.

The force applied on the fluid element : $F = PA$

The work done in pushing the fluid element into the control volume:

i.e. the flow work : $W_{flow} = FL = PAL = PV$ **(kJ)**

On a unit mass basis $w_{flow} = Pv$ **(kJ/kg)**



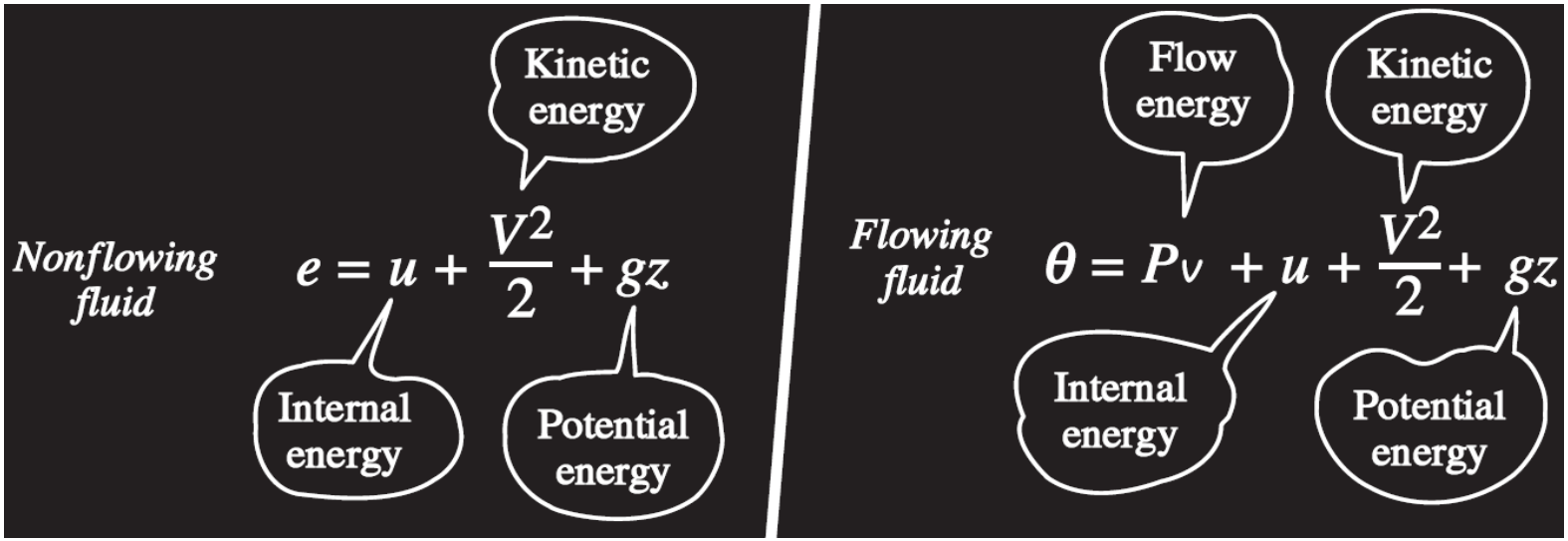
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Total Energy of a flowing fluid

For a nonflowing fluid : e = u + ke + pe = u + $\frac{V^2}{2}$ +gz (kJ/kg)

For a flowing fluid the total energy θ = Pv + e = $\underbrace{\text{Pv + u}}_{\text{h}}$ + $\frac{V^2}{2}$ + gz (kJ/kg)

Hence θ = h + ke + pe = h + $\frac{V^2}{2}$ + gz (kJ/kg)

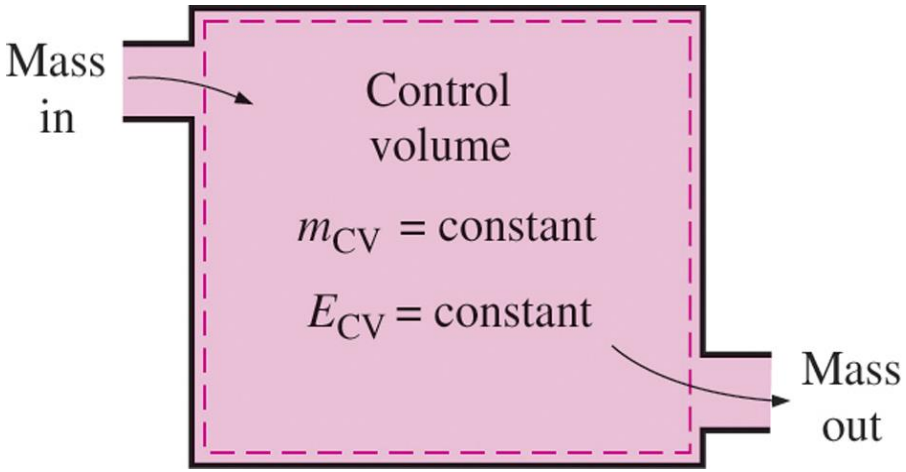


The total energy consists of three parts for a nonflowing fluid and four parts for a flowing fluid.

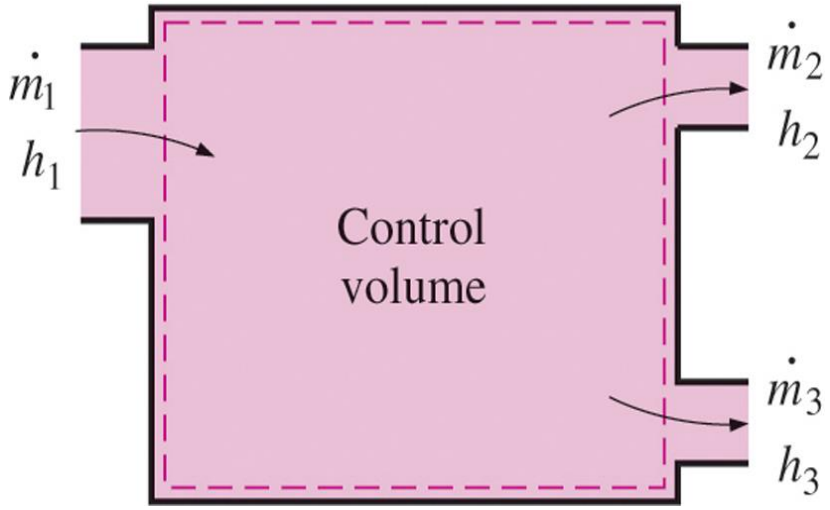
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THE STEADY-FLOW PROCESS

Steady flow is defined such that all properties at each point in a system remain constant with respect to time. (e.g. turbines, compressors, and heat exchangers)



Under steady-flow conditions, the mass and energy contents of a control volume remain constant.



Under steady-flow conditions, the fluid properties at an inlet or exit remain constant (do not change with time).

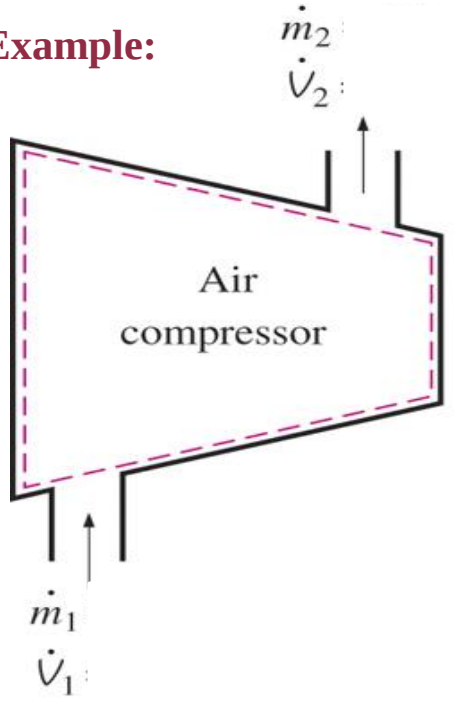
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Conservation of mass

m_{cv} = constant for steady-flow process

$$\sum \dot{m}_i = \sum \dot{m}_e$$

Example:



If there is only one inlet and one exit

$$\dot{m}_1 = \dot{m}_2 \text{ (kg/s) or } \rho_1 V_1 A_1 = \rho_2 V_2 A_2 \qquad \text{(or } \frac{1}{v_1} V_1 A_1 = \frac{1}{v_2} V_2 A_2 \text{)}$$

where $\rho = \text{density (kg/m}^3\text{)}$

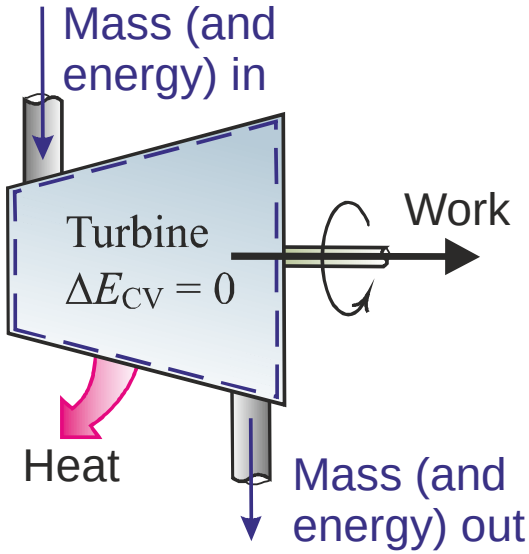
$v = \text{specific volume (m}^3\text{/kg)}$

$V = \text{average velocity (m/s)}$

$A = \text{cross-sectional area normal to area (m}^2\text{)}$

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Conservation of energy for steady flow



$$E_{CV} = \text{constant} \quad \text{or} \quad \Delta E_{CV} = 0$$

$$\dot{Q} - \dot{W} = \sum \dot{E}_{out} - \sum \dot{E}_{in} + \Delta \dot{E}_{cv}$$

$$\Rightarrow \dot{Q} - \dot{W} = \sum \dot{m}_e \theta_e - \sum \dot{m}_i \theta_i$$

$$\Rightarrow \dot{Q} - \dot{W} = \sum \dot{m}_e \left(h_e + \frac{V_e^2}{2} + g z_e\right) - \sum \dot{m}_i \left(h_i + \frac{V_i^2}{2} + g z_i\right)$$

For one-inlet, one-exit systems(i.e single stream systems):

$$\dot{m}_1 = \dot{m}_2$$

$$\dot{Q} - \dot{W} = \dot{m}\{\underbrace{h_2 - h_1}_{\Delta h} + \underbrace{\frac{V_2 - V_1}{2}}_{\Delta ke} + g\underbrace{(z_2 - z_1)}_{\Delta pe}\}$$

In many cases:

$$\Delta ke = \Delta pe = 0$$

$$\Rightarrow q - w = \Delta h$$

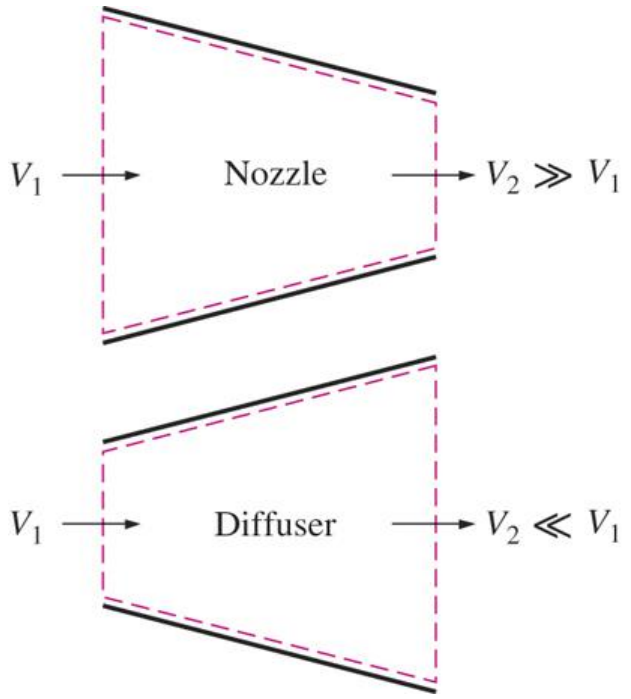
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Some Steady-Flow Engineering Devices

1.Nozzels and Diffusers

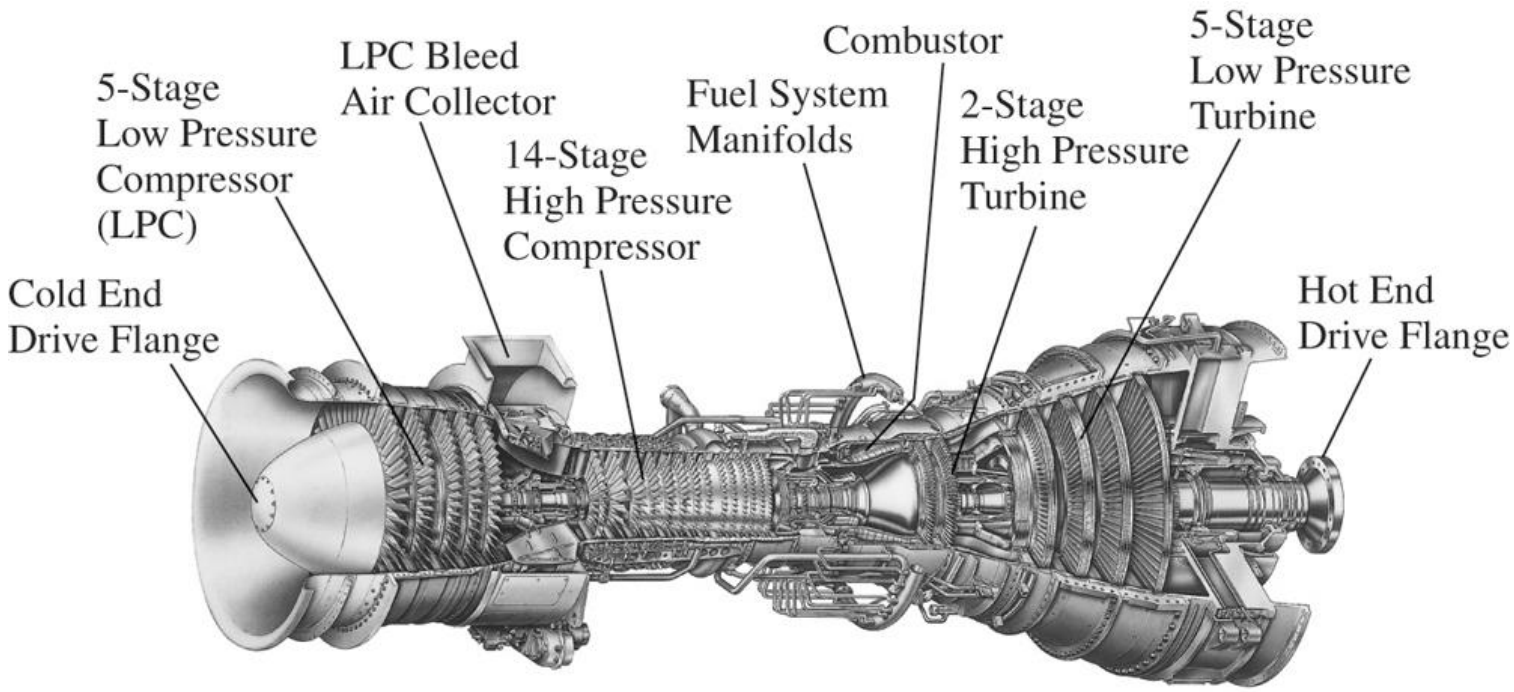
$$\overset{0}{\dot{q}} - \overset{0}{\dot{w}} = \Delta h + \Delta ke + \overset{0}{\Delta pe}$$

$$0 = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2}$$



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2.Turbines and compressors



A modern land-based gas turbine used for electric power production. This is a General Electric LM5000 turbine. It has a length of 6.2 m, it weighs 12.5 tons, and produces 55.2 MW at 3600 rpm with steam injection.

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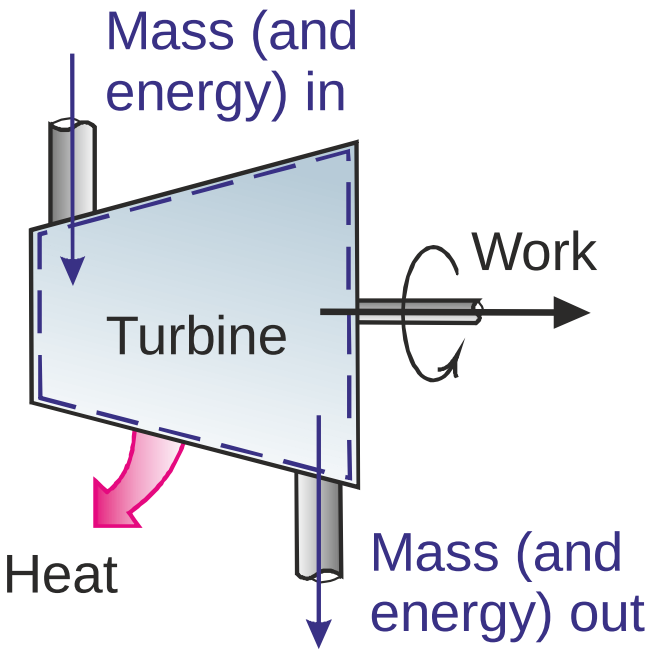
2.Turbines and compressors

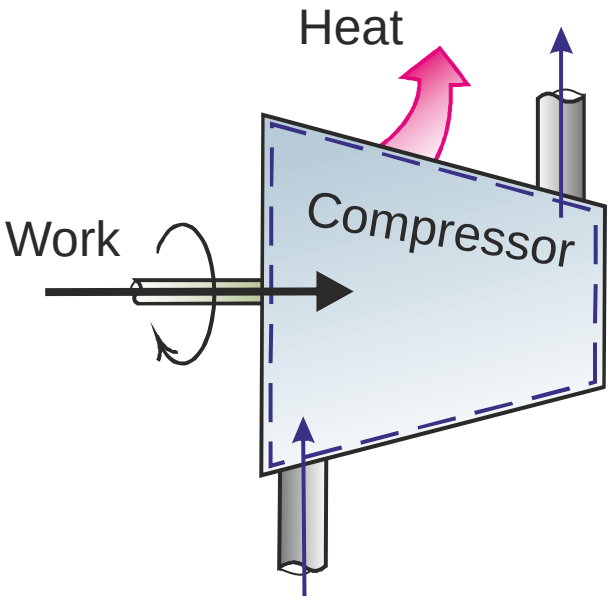
$$q - w = \Delta h + \Delta ke + \Delta pe$$

$$w = -(h_2 - h_1 + \frac{V_2^2 - V_1^2}{2})$$

$$q - w = \Delta h + \cancel{\Delta ke}^{0} + \cancel{\Delta pe}^{0}$$

$$q - w = h_2 - h_1$$





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Ex: Steam turbine

For steady flow

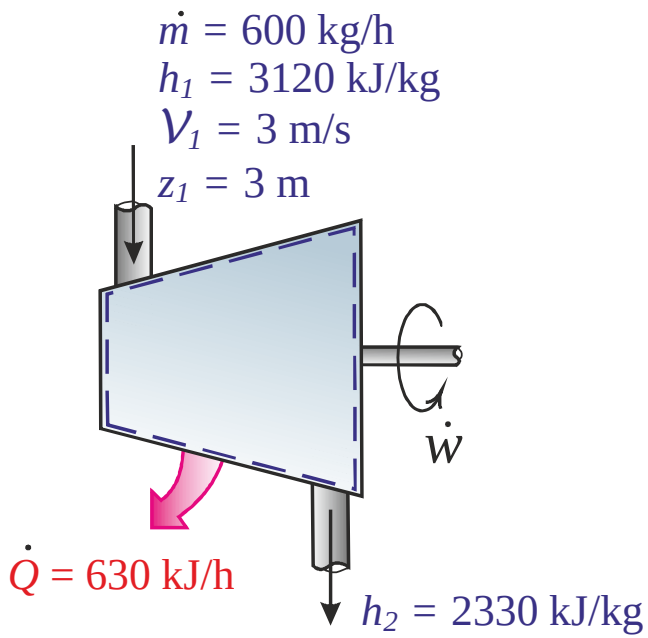
$\dot{m}_1 = \dot{m}_2 = \dot{m}$

$$\dot{Q} - \dot{W} = \dot{m}\{h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\}$$

$$\frac{\dot{Q}}{\dot{m}} - \frac{\dot{W}}{\dot{m}} + (h + \frac{V^2}{2} + gz)_1 - (h + \frac{V^2}{2} + gz)_2 = 0$$

$$\dot{m} = \frac{600\,{}^{kg}/_{hr}}{3600\,{}^{\sec}/_{hr}} = 0.167\,{}^{kg}/_{\sec}$$

$$\dot{Q} = \frac{630\,{}^{kg}/_{hr}}{3600\,{}^{\sec}/_{hr}} = 0.175kW$$



$$-\frac{0.175\,{}^{kJ}/_{s}}{0.167\,{}^{kg}/_{s}} - \frac{\dot{W}}{0.167\,{}^{kg}/_{s}} + (3120\,{}^{kJ}/_{kg} + \frac{(100\,{}^{m}/_{s})^2}{2(1000\,{}^{J}/_{kJ})} + \frac{9.81\,{}^{m}/_{s^2}}{1000\,{}^{J}/_{kJ}}) - (2330\,{}^{kJ}/_{kg} + \frac{(130\,{}^{m}/_{s})^2}{2(1000\,{}^{J}/_{kJ})} + 0) = 0$$

$$\Rightarrow -1.05\,{}^{kJ}/_{kg} - \frac{\dot{W}}{0.167\,{}^{kg}/_{s}} + (3120\,{}^{kJ}/_{kg} + 5\,{}^{kJ}/_{kg} + 0.029\,{}^{kJ}/_{kg}) - (2330 + 8.45 + 0) = 0$$

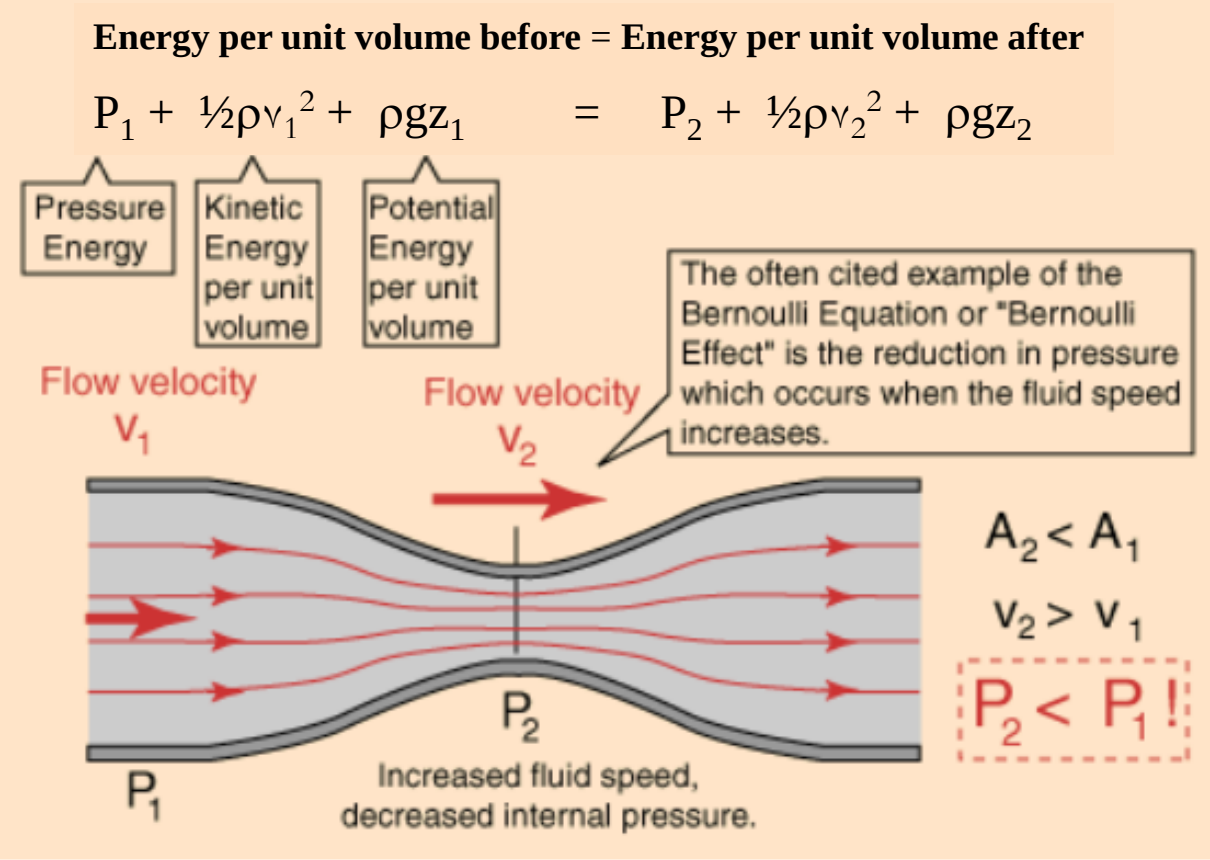
$$\Rightarrow \dot{W} = +131.183kW$$

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3.Throling Valves

Throttling valves are *any kind of flow-restricting devices* that cause a significant pressure drop in the fluid.

Why there is a pressure drop?



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3.Throling Valves

What is the difference between a turbine and a throttling valve?

The pressure drop in the fluid is often accompanied by a *large drop in temperature*, and for that reason throttling devices are commonly used in refrigeration and air-conditioning applications.

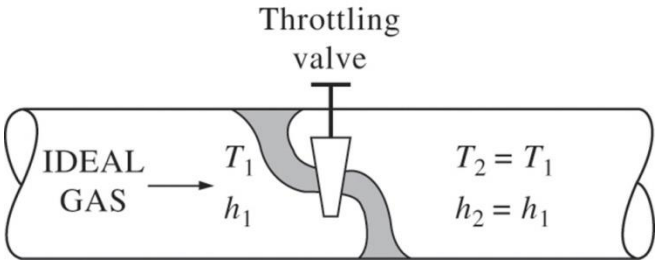
→ Energy balance

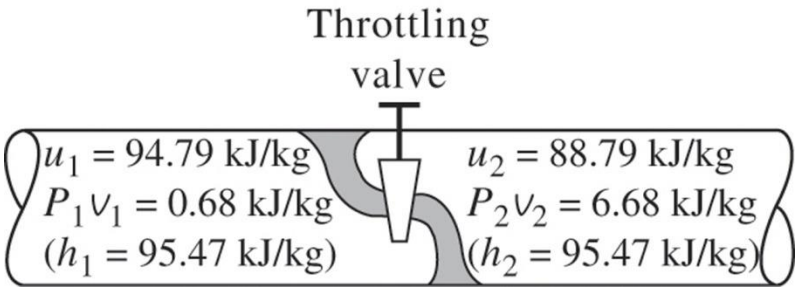
$$\cancel{q}^{0} - \cancel{w}^{0} = \Delta h + \Delta \cancel{ke}^{0} + \Delta \cancel{pe}^{0}$$

$$\Rightarrow h_2 = h_1$$

$$u_1 + P_1 v_1 = u_2 + P_2 v_2$$

Internal energy + Flow energy = Constant

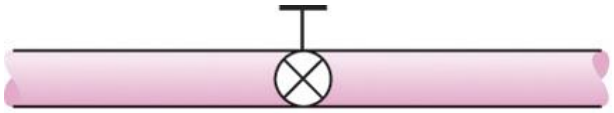




During a throttling process, the enthalpy of a fluid remains constant. But internal and flow energies may be converted to each other.

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3.Throttling Valves



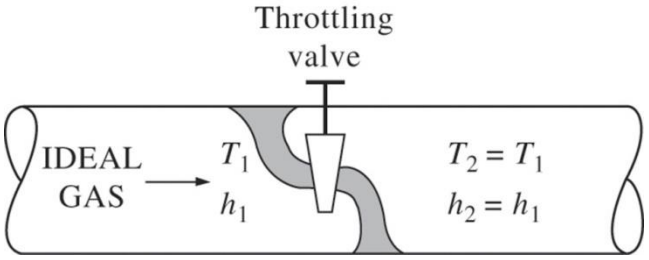
(*a*) An adjustable valve



(*b*) A porous plug



(*c*) A capillary tube



$$h_2 \cong h_1$$

$$u_1 + P_1 v_1 = u_2 + P_2 v_2$$

Internal energy +flow =const.

If $P_2 v_2$ increase then u_2 decreases $\longrightarrow$ drop in temperature

If $P_1 v_1$ decrease then u_1 increases $\longrightarrow$ increase in temperature

For an ideal gas, h=h(T) and since $h_1 = h_2$, the temperature does not change $T_1 = T_2$.

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4. Heat exchangers

Mass flow rate of fluid A:

Mass flow rate of fluid B:

$$\dot{m}_i = \dot{m}_e = \dot{m}_A$$

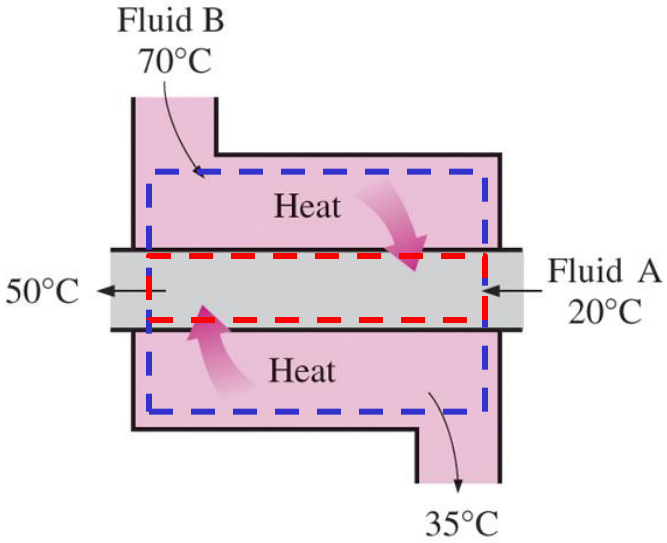
$$\dot{m}_i = \dot{m}_e = \dot{m}_B$$

Energy equation for blue CV:

$$\dot{Q} - \dot{W} = \sum \dot{m}_e \left(h_e + \frac{V_e^2}{2} + g z_e\right) - \sum \dot{m}_i \left(h_i + \frac{V_i^2}{2} + g z_i\right)$$

$$\sum \dot{m}_i h_i = \sum \dot{m}_e h_e$$

$$\dot{m}_B h_{Bi} + \dot{m}_A h_{Ai} = \dot{m}_B h_{Be} + \dot{m}_A h_{Ae}$$



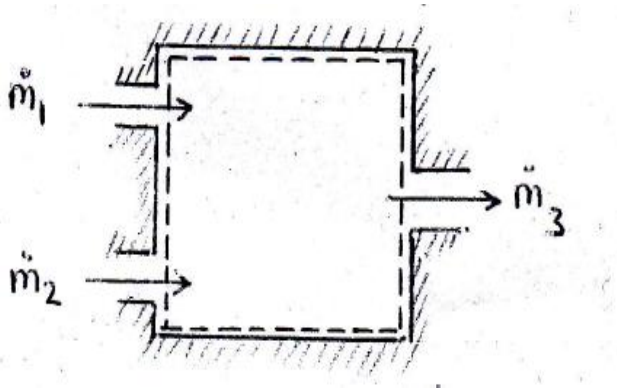
Energy equation for red CV: $\longrightarrow$ single stream

$$\dot{Q} - \dot{W}^{\,0} = \dot{m}_A (\Delta h + \Delta ke^{\,0} + \Delta pe^{\,0})$$

$$\dot{Q} = \dot{m}_A (h_2 - h_1)$$

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5.Mixing Chambers



$$\sum \dot{m}_i = \sum \dot{m}_2$$

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3 \rightarrow \frac{\dot{m}_1}{\dot{m}_2} + 1 = \frac{\dot{m}_3}{\dot{m}_2}$$

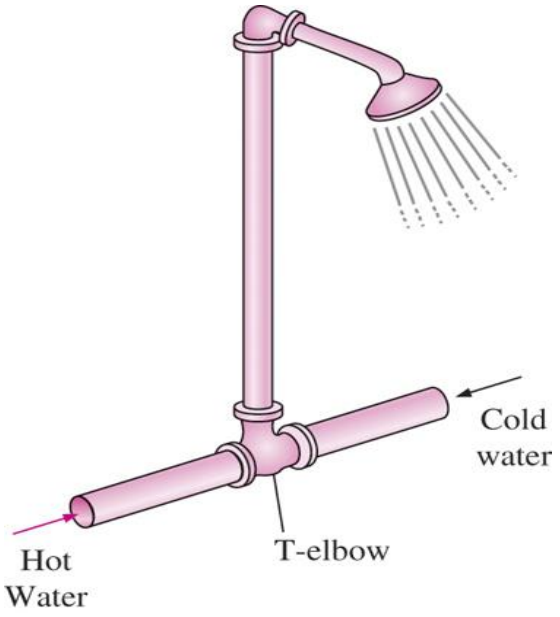
$$\dot{Q} - \dot{W} = \sum \dot{m}_e \left(h_e + \frac{V_e^2}{2} + g z_e\right) - \sum \dot{m}_i \left(h_i + \frac{V_i^2}{2} + g z_i\right)$$

$$\sum \dot{m}_i h_i = \sum \dot{m}_e h_e$$

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

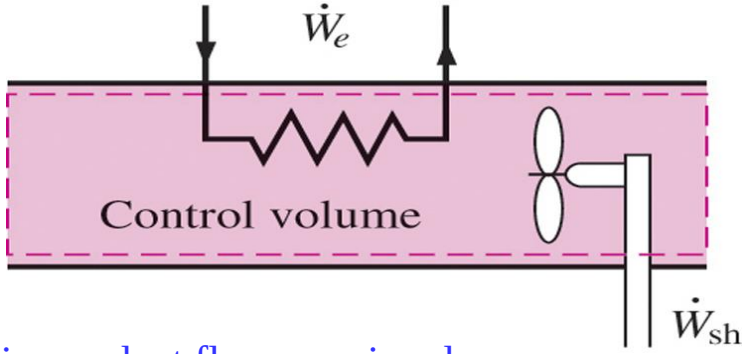
Divide by $\dot{m}_2$ and let $y = \frac{\dot{m}_1}{\dot{m}_2} \therefore y\, h_1 + h_2 = (y+1)\, h_3$

The T-elbow of an ordinary shower serves as the mixing chamber for the hot- and the cold-water streams.



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6.Pipe and Duct Flow

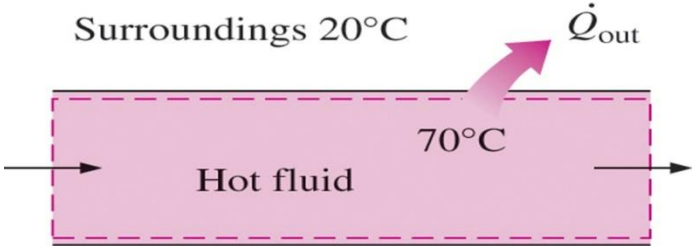


Pipe or duct flow may involve more than one form of work at the same tim

$$\dot{Q} - \dot{W} = \dot{m}\,(\Delta h + \Delta ke + \Delta pe)$$

$$\dot{Q} - \dot{W} = \dot{m}(h_2 - h_1)$$

When air treated as ideal gas



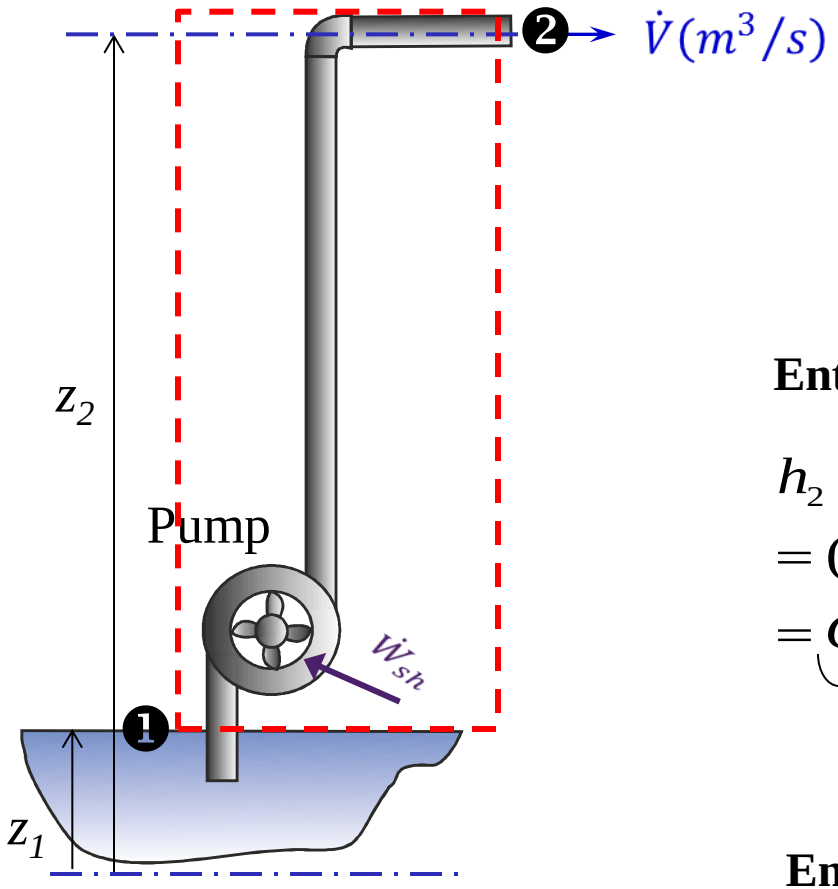
Heat losses from a hot fluid flowing through an uninsulated pipe or duct to the cooler environment may be very significant.

$$v_1 = \frac{RT_1}{P_1}$$

$$\dot{m} = \frac{\dot{V}_1}{v_1}$$

$$\Rightarrow \dot{Q} - \dot{W} = \dot{m}C_p(T_2 - T_1)$$

$$(-\dot{W}_e - W_{sh})$$



$$\dot{m} = \rho \dot{V} \qquad (kg/s)$$

$$V_1 = \frac{\dot{m}}{\rho_1 A_1} \qquad (m/s)$$

$$V_2 = \frac{\dot{m}}{\rho_2 A_2} \qquad (m/s)$$

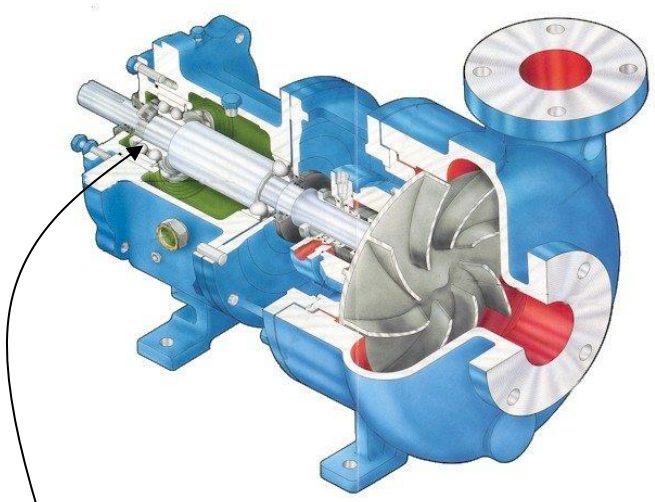
Enthalpy change for incompressible substance:

$$h_2 - h_1 = (u_2 + P_2 v_2) - (u_1 + P_1 v_1)$$
$$= (u_2 - u_1) + v(P_2 - P_1)$$
$$= \underbrace{C(T_2 - T_1)}_{\Delta u} + v(P_2 - P_1)$$

Energy equation becomes:

$$\dot{Q} - \dot{W}_{sh} = \dot{m}\{ C\,(T_2 - T_1) + v\,(P_2 - P_1) + \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2} + g\,(z_2 - z_1)\}$$

Energy Equation: $\dot{Q} - \dot{W}_{sh} = \dot{m}\left\{C(T_2 - T_1) + v(P_2 - P_1) + \frac{{\vartheta_2}^2 - {\vartheta_1}^2}{2} + g(z_2 - z_1)\right\}$



Electric Pump

When $\dot{Q}$ is neglible and $T_1 = T_2$:

$$\rightarrow \dot{W}_{sh} = \dot{m}\left\{v(P_2 - P_1) + \frac{{\vartheta_2}^2 - {\vartheta_1}^2}{2} + g(z_2 - z_1)\right\}$$

In the case of negligible changes of kinetic and potential energies:

$$\rightarrow \dot{W}_{sh} \approx \dot{V}\{(P_2 - P_1)\}$$

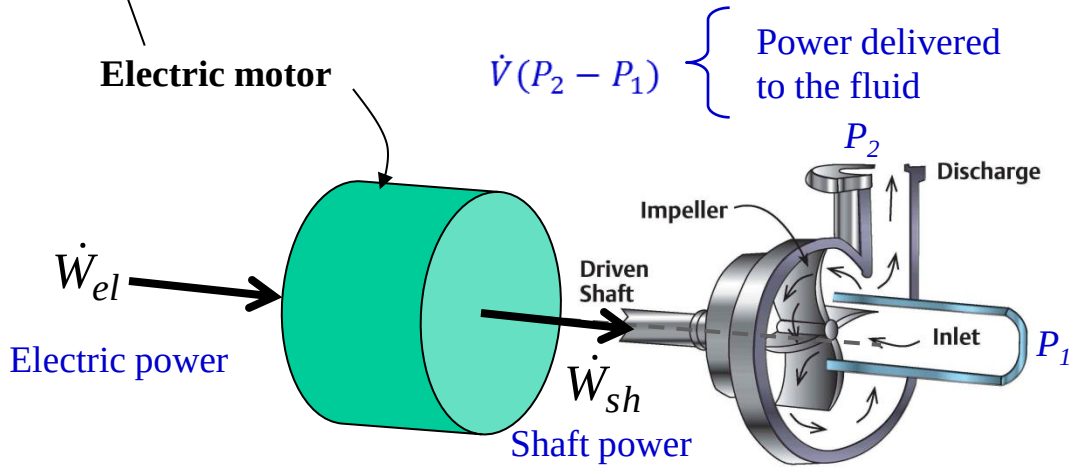
Definition of pump efficiency:

$$\eta_{pump} = \frac{\text{Water (output) Power}}{\text{Shaft (input) Power}} = \frac{\dot{V}(\frac{\text{m}^3}{\text{s}}) \times (P_2 - P_1)\text{kPa}}{\dot{W}_{sh}(\text{kW})}$$

$$\eta_{motor} = \dot{W}_{sh} \Big/ \dot{W}_{el}$$

The overall pump efficiency:

$$\eta_{ov} = \eta_{pump} \times \eta_{motor}$$



Electric power consumption:

$$\dot{W}_{el} = \frac{\dot{W}_{sh}}{\eta_{motor}} = \frac{\dot{V} \times (P_2 - P_1)}{\eta_{motor}\eta_{pump}}$$

The Second Law of Thermodynamics

The second law of thermodynamics asserts that processes occur in a certain direction and that the energy has *quality* as well as *quantity*.

The first law places no restriction on the direction of a process, and satisfying the first law does not guarantee that the process will occur. Thus, we need another general principle (second law) to identify whether a process can occur or not.

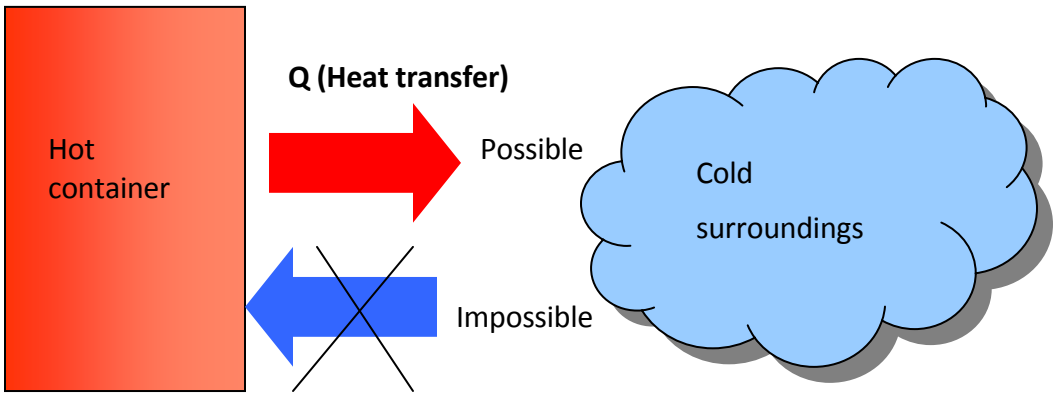


Fig. 1: Heat transfer from a hot container to the cold surroundings is possible; however, the reveres process (although satisfying the first law) is impossible.

A process can occur when and only when it satisfies both the first and the second laws of thermodynamics.

The second law also asserts that energy has a quality. Preserving the quality of energy is a major concern of engineers. In the above example, the energy stored in a hot container (higher temperature) has higher quality (ability to work) in comparison with the energy contained (at lower temperature) in the surroundings.

The second law is also used in determining the theoretical limits for the performance of commonly used engineering systems, such as heat engines and refrigerators etc.

Thermal Energy Reservoirs

Thermal energy reservoirs are hypothetical bodies with a *relatively* large thermal energy capacity (mass x specific heat) that can supply or absorb finite amounts of heat *without undergoing any change in temperature*. Lakes, rivers, atmosphere, oceans are example of thermal reservoirs.

A two-phase system can be modeled as a reservoir since it can absorb and release large quantities of heat while remaining at constant temperature.

A reservoir that supplies energy in the form of heat is called a *source* and one that absorbs energy in the form of heat is called a *sink*.

Heat Engines

Heat engines convert heat to work. There are several types of heat engines, but they are characterized by the following:

1- They all receive heat from a high-temperature source (oil furnace, nuclear reactor, etc.)

2- They convert part of this heat to work

3- They reject the remaining waste heat to a low-temperature sink

4- They operate in a cycle.

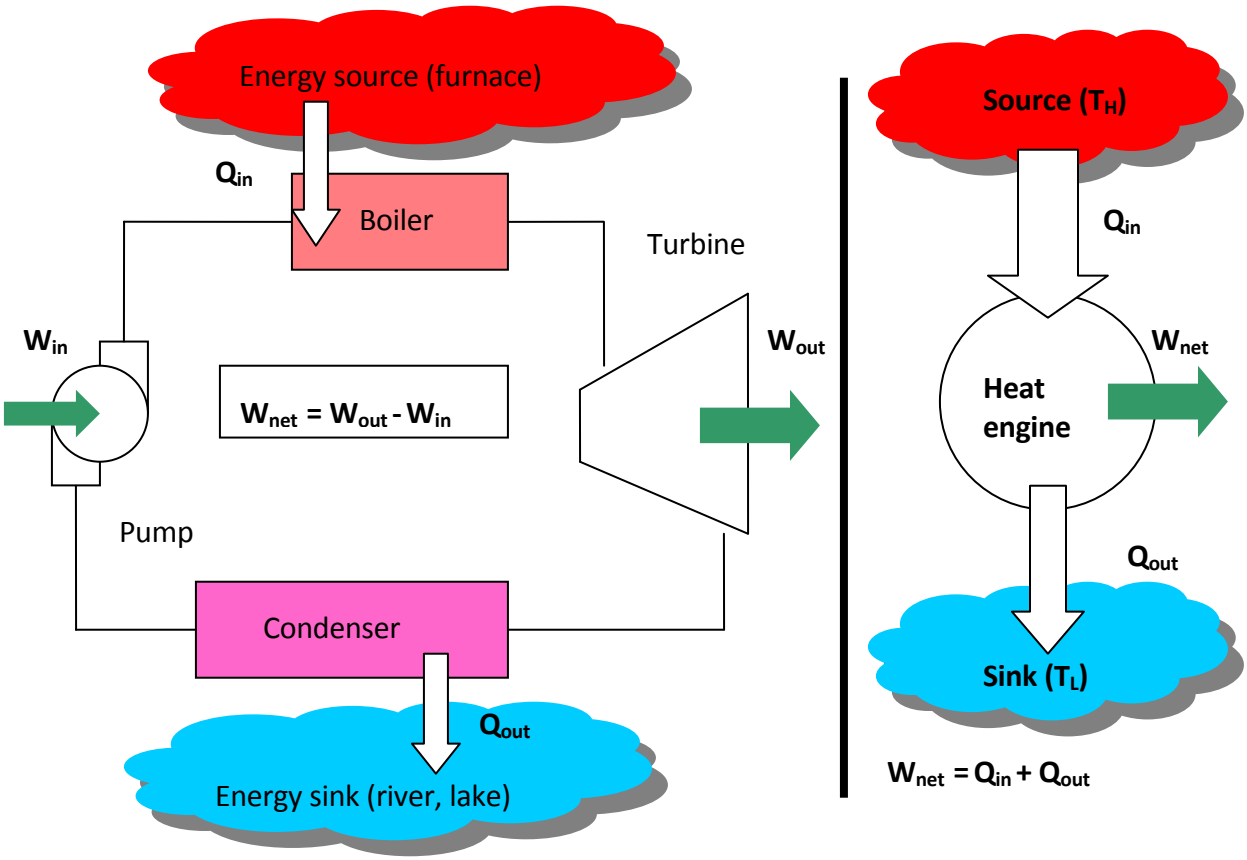


Fig. 2: Steam power plant is a heat engine.

Thermal efficiency: is the fraction of the heat input that is converted to the net work output (efficiency = benefit / cost).

$$\eta_{th} = \frac{W_{net,out}}{Q_{in}} \qquad and \qquad W_{net,out} = Q_{in} - Q_{out}$$

$$\eta_{th} = 1 - \frac{Q_{out}}{Q_{in}}$$

The thermal efficiencies of work-producing devices are low. Ordinary spark-ignition automobile engines have a thermal efficiency of about 20%, diesel engines about 30%, and power plants in the order of 40%.

Is it possible to save the rejected heat Q_{out} in a power cycle? The answer is NO, because without the cooling in condenser the cycle cannot be completed. Every heat engine *must* waste some energy by transferring it to a *low-temperature* reservoir in order to complete the cycle, *even in idealized cycle*.

The Second Law: Kelvin-Planck Statement

It is impossible for any device that operates on a cycle to receive heat from a single reservoir and produce a net amount of work. In other words, no heat engine can have a thermal efficiency of 100%.

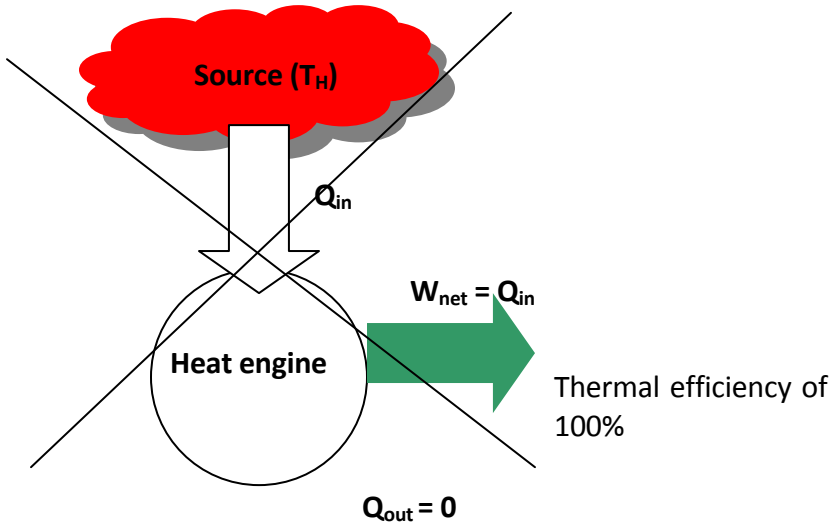


Fig.3: A heat engine that violates the Kelvin-Planck statement of the second law cannot be built.

Refrigerators and Heat Pumps

In nature, heat flows from high-temperature regions to low-temperature ones. The reverse process, however, cannot occur by itself. The transfer of heat from a low-temperature region to a high-temperature one requires special devices called *refrigerators*. Refrigerators are cyclic devices, and the working fluids used in the cycles are called *refrigerant*.

Heat pumps transfer heat from a low-temperature medium to a high-temperature one. Refrigerators and heat pumps are essentially the same devices; they differ in their objectives only. Refrigerator is to maintain the refrigerated space at a low temperature. On the other hand, a heat pump absorbs heat from a low-temperature source and supplies the heat to a warmer medium.

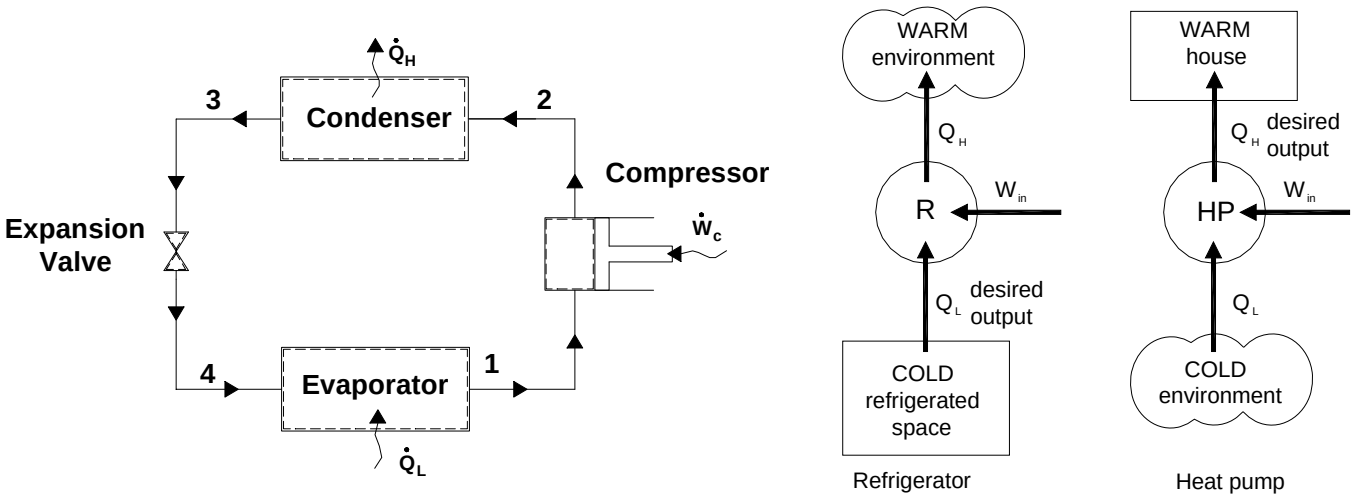


Fig. 4: Objectives of refrigerator and heat pump.

Coefficient of Performance (COP)

The performance of refrigerators and heat pumps is expressed in terms of the coefficient of performance (COP) which is defined as

$$COP_R = \frac{Benefit}{Cost} = \frac{q_L}{w_c} \qquad COP_{HP} = \frac{Benefit}{Cost} = \frac{q_H}{w_c}$$

It can be seen that

$$COP_{HP} = COP_R + 1$$

Air conditioners are basically refrigerators whose refrigerated space is a room or a building.

The Energy Efficiency Rating (EER): is the amount of heat removed from the cooled space in BTU's for 1 Wh (watt-hour)

$$EER = 3.412\ COP_R$$

Most air conditioners have an EER between 8 to 12 (COP of 2.3 to 3.5).

The Second Law of Thermodynamics: Clausius Statement

It is impossible to construct a device that operates in a cycle and produces no effect other than the transfer of heat from a lower-temperature body to higher-temperature body. In other words, a refrigerator will not operate unless its compressor is driven by an external power source.

Kelvin-Planck and Clausius statements of the second law are negative statements, and a negative statement cannot be proved. So, the second law, like the first law, is based on experimental observations.

The two statements of the second law are equivalent. In other words, any device violates the Kelvin-Planck statement also violates the Clausius statement and vice versa.

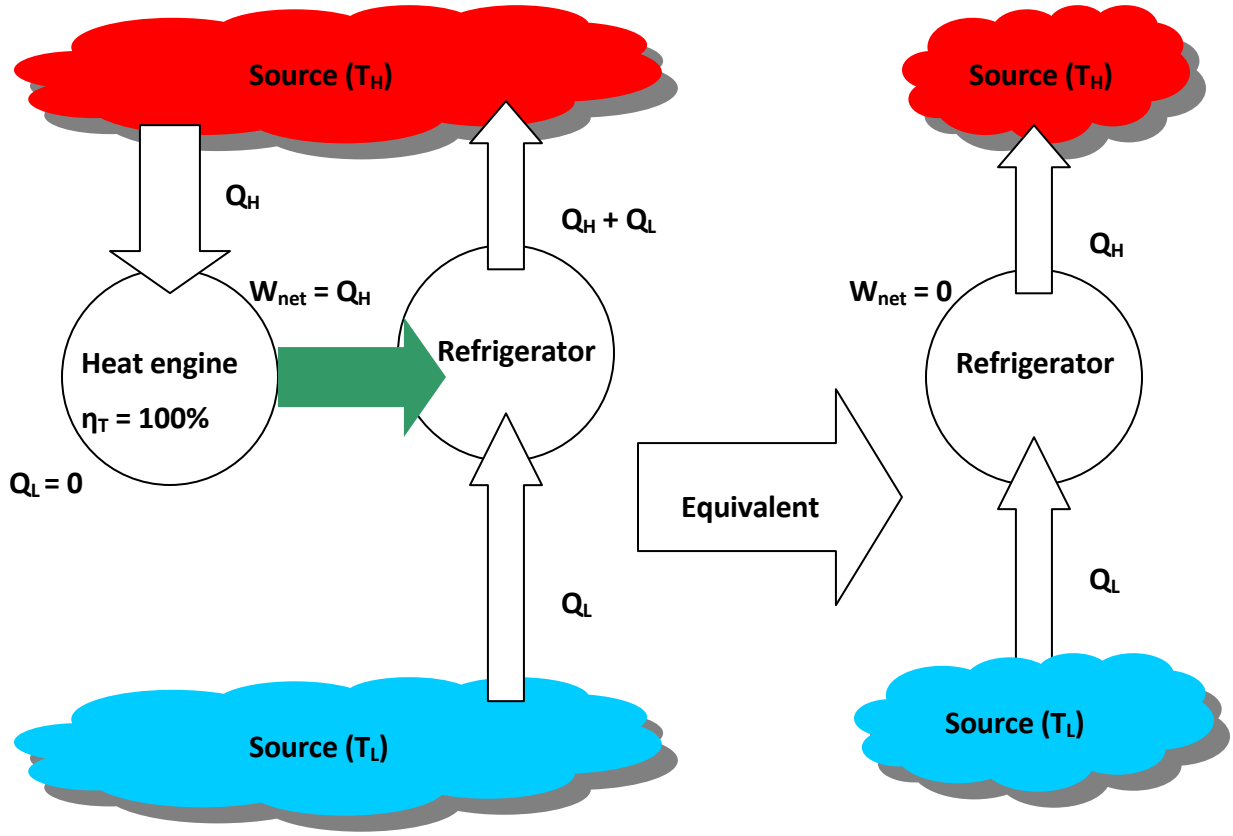


Fig. 5: The violation of the Kelvin-Planck statement leads to violation of Clausius.

Any device that violates the first law of thermodynamics (by creating energy) is called a *perpetual-motion machine of the first kind* (PMM1), and the device that violates the second law is called a *perpetual-motion machine of the second kind* (PMM2).

Reversible and Irreversible Process

A *reversible* process is defined as a process that can be reversed without leaving any trace on the surroundings. It means both system and surroundings are returned to their initial states at the end of the reverse process. Processes that are not reversible are called *irreversible*.

Reversible processes do not occur and they are only idealizations of actual processes. We use reversible process concept because, a) they are easy to analyze (since system passes through a series of equilibrium states); b) they serve as limits (idealized models) to which the actual processes can be compared.

Some factors that cause a process to become irreversible:

- Friction
- Unrestrained expansion and compression

• mixing

• Heat transfer (finite ΔT)

• Inelastic deformation

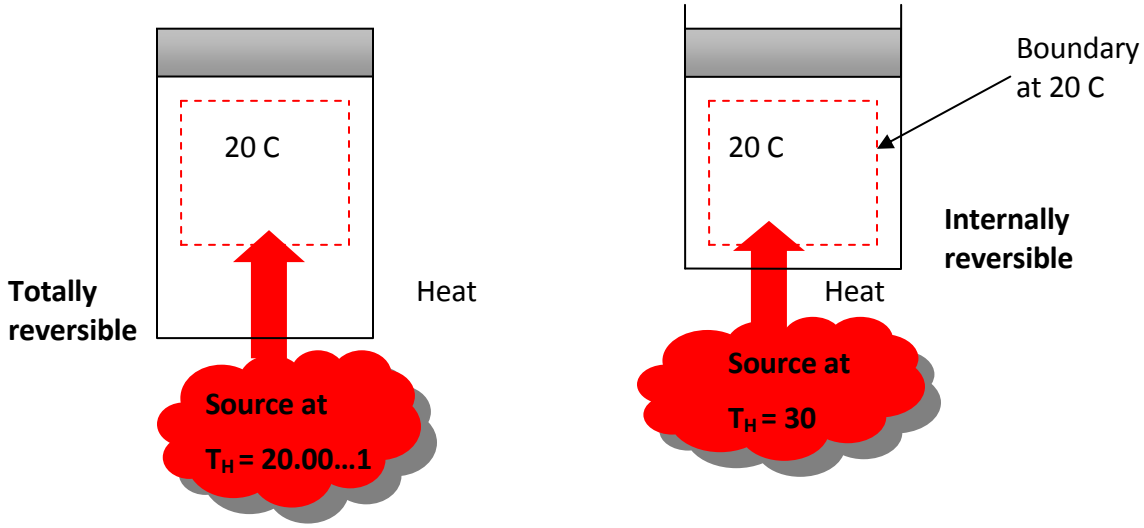
• Chemical reactions

In a reversible process things happen very slowly, without any resisting force, without any space limitation → everything happens in a highly organized way (it is not physically possible - it is an idealization).

Internally reversible process: if no irreversibilities occur within the boundaries of the system. In these processes a system undergoes through a series of equilibrium states, and when the process is reversed, the system passes through exactly the same equilibrium states while returning to its initial state.

Externally reversible process: if no irreversibilities occur outside the system boundaries during the process. Heat transfer between a reservoir and a system is an externally reversible process if the surface of contact between the system and reservoir is at the same temperature.

Totally reversible (reversible): both externally and internally reversible processes.



The Carnot Cycle

The efficiency of a heat-engine cycle greatly depends on how the individual processes that make up the cycle are executed. The net work (or efficiency) can be maximized by using reversible processes. The best known reversible cycle is the *Carnot cycle*.

Note that the reversible cycles cannot be achieved in practice because of irreversibilities associated with real processes. But, the reversible cycles provide upper limits on the performance of real cycles.

Consider a gas in a cylinder-piston (closed system). The Carnot cycle has four processes:

1-2 Reversible isothermal expansion: The gas expands slowly, doing work on the surroundings. Reversible heat transfer from the heat source at T_H to the gas which is also at T_H.

2-3 Reversible adiabatic expansion: The cylinder-piston is now insulated (adiabatic) and gas continues to expand reversibly (slowly). So, the gas is doing work on the surroundings, and as a result of expansion the gas temperature reduces from T_H to T_L.

3-4: Reversible isothermal compression: The gas is allowed to exchange heat with a sink at temperature T_L as the gas is being slowly compressed. So, the surroundings is doing work (reversibly) on the system and heat is transferred from the system to the surroundings (reversibly) such that the gas temperature remains constant at T_L.

4-1: Reversible adiabatic compression: The gas temperature is increasing from T_L to T_H as a result of compression.

Carnot cycle is the most efficient cycle operating between two specified temperature limits.

The efficiency of all reversible heat engines operating between the two same reservoirs are the same.

The thermal efficiency of a heat engine (reversible or irreversible) is:

$$\eta_{th} = 1 - \frac{Q_L}{Q_H}$$

For the Carnot cycle, it can be shown:

$$\eta_{th,Carnot} = 1 - \frac{T_L}{T_H}$$

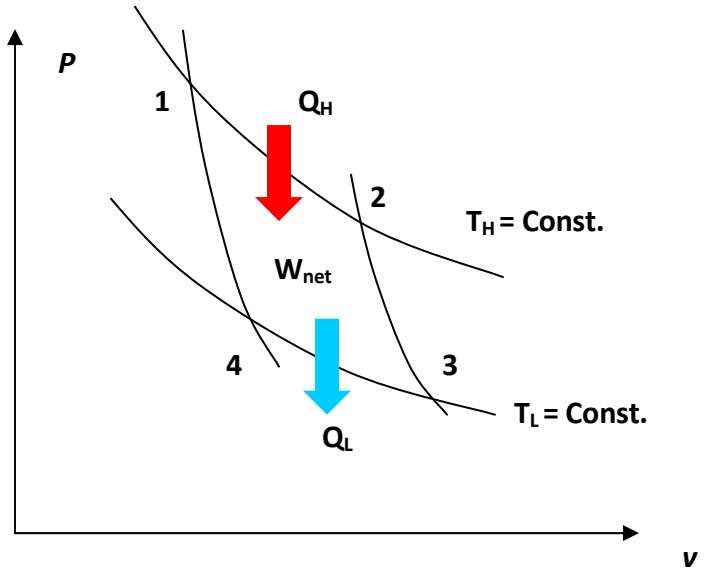


Fig. 6: P-v diagram for the Carnot cycle.

The efficiency of an irreversible (real) cycle is always less than the efficiency of the Carnot cycle operating between the same two reservoirs.

$$\eta_{th} = \begin{cases} < \eta_{th,rev} & \text{irreversible heat engine} \\ = \eta_{th,rev} & \text{reversible heat engine} \\ > \eta_{th,rev} & \text{impossible heat engine!} \end{cases}$$

Consider a Carnot heat engine working between two thermal reservoirs T_L = 300 K and T_H. The thermal efficiency of the heat engine increases as the heat source temperature T_H is increased.

T_H K	η_{th} %
1000	70
900	66.6
500	40
350	14.3

The thermal efficiency of actual heat engine can be maximized by supplying heat to the engine at the highest possible temperature (limited by material strength) and rejecting heat to lowest possible temperature (limited by the cooling medium temperature such as atmosphere, lake, river temperature).

From the above table, it can also be seen that the energy has a quality. More of the high-temperature thermal energy can be converted to work. Therefore, the higher the temperature, the higher the quality of the energy will be.

The Carnot Refrigeration and Heat Pump Cycle

A refrigerator or heat pump that operates on the reverse Carnot cycle is called a *Carnot Refrigerator*, or a *Carnot heat pump*.

The Coefficient of performance of any refrigerator or heat pump (reversible or irreversible) is given by:

$$COP_R = \frac{1}{Q_H / Q_L - 1} \quad \text{and} \quad COP_{HP} = \frac{1}{1 - Q_L / Q_H}$$

COP of all reversible refrigerators or heat pumps can be determined from:

$$COP_{R,rev} = \frac{1}{T_H / T_L - 1} \quad \text{and} \quad COP_{HP,rev} = \frac{1}{1 - T_L / T_H}$$

Also, similar to heat engine, one can conclude:

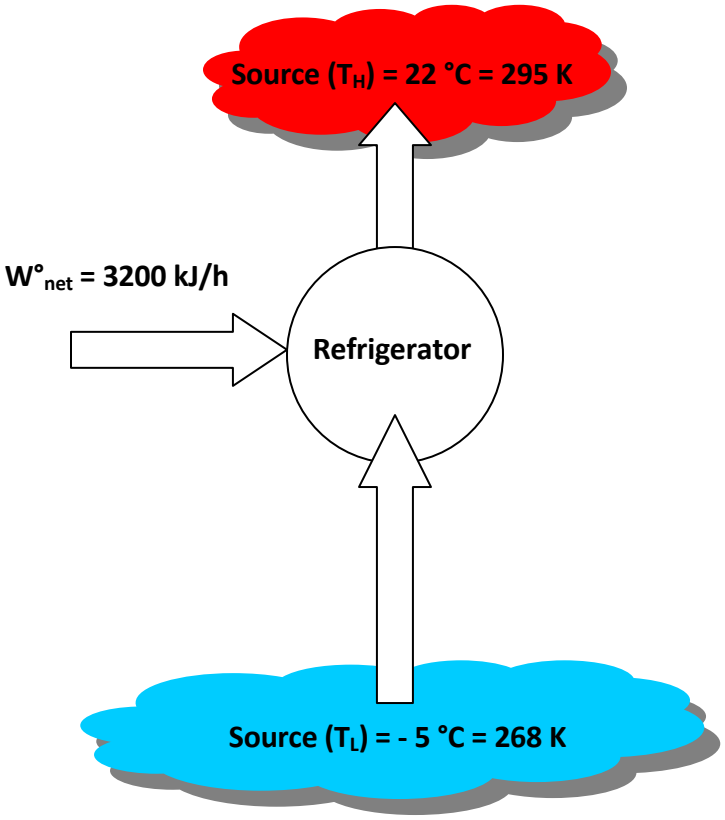
$$COP_R = \begin{cases} < COP_{R,rev} & \text{irreversible refrigerator} \\ = COP_{th,rev} & \text{reversible refrigerator} \\ > COP_{th,rev} & \text{impossible refrigerator!} \end{cases}$$

Example 1: Refrigerator Performance

A refrigerator maintains the temperature of the freezer compartment at -5 C when the air surrounding the refrigerator is at 22 °C. The rate of heat transfer from the freezer compartment to the refrigerant (the working fluid) is 8000 kJ/h and the power input required to operate the refrigerator is 3200 kJ/h. Determine the coefficient of performance of the refrigerator and compare with the coefficient of performance of a reversible refrigeration cycle operating between reservoirs at the same temperatures.

Assumptions:

- Steady-state operation of the refrigerator.
- The freezer compartment and the surrounding air play the roles of the cold and hot reservoirs, respectively.



$$COP_R = Q^\circ_C / W^\circ_{cycle}$$

$$COP_R = 8000\ (kJ/h) / 3200\ (kJ/h) = 2.5$$

The coefficient of performance of a Carnot refrigerator working between the same two reservoirs is:

$$COP_{R,Carnot} = \frac{1}{T_H / T_C - 1} = \frac{1}{295/268 - 1} = 9.9$$

CHAPTER 3

Carnot Cycle

Power Cycles

Most power producing devices operate on cycles. To make simple thermodynamics analysis possible, we use the concept of ideal cycle. These idealized cycles serve as starting point for more in-depth studies.

Heat engines are designed to convert thermal energy to work. The thermal efficiency, η_{th}, defines as:

$$\eta_{th} = \frac{Benefit}{Cost} = \frac{W_{net}}{Q_{in}} \quad \text{or} \quad \eta_{th} = \frac{w_{net}}{q_{in}}$$

Ideal cycles are internally reversible, but unlike the Carnot cycle, they are not necessarily externally reversible. Thus, the thermal efficiency of an ideal cycle, in general, is less than that of a totally reversible cycle operating between the same temperature limits, i.e., Carnot cycle. Therefore, nobody can develop a cycle more efficient than the Carnot cycle.

Common Assumptions in Idealizations Power Cycles

1- The cycles do not have any friction. Thus, no pressure drops in the working fluid.

2- All expansions and compressions are considered quasi-equilibrium.

3- No heat losses in pipes and other components.

4- Changes in kinetic and potential energies are negligible. Except for nozzles and diffusers which are specifically designed to change the velocity of passing fluid.

T-s and P-v diagrams are useful as visual aids in the analysis of ideal power cycle.

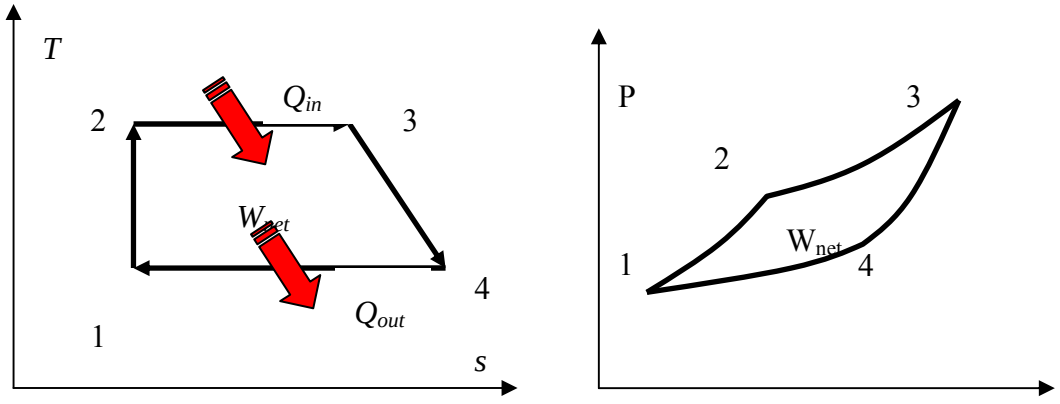


Fig. 2-1: Areas enclosed by the process curves in T-s and P-v diagrams represent the net work of the idealized cycle.

Remember that an ideal power cycle does not involve any internal irreversibilities; thus one can conclude:

1- On a T-s diagram, a *heat addition* process proceeds in the direction of increasing entropy and vice versa.

2- The area under the process curve on a T-s diagram represents the heat transfer for that process.

3- The area enclosed by the process curve on both T-s and P-v diagrams represent the net work of the cycle.

4- The ratio of the area enclosed by the cyclic curve to the area under the heat-addition process curve represents the thermal efficiency of the cycle.

5- On a T-s diagram, a vertical line represents an isentropic process (no heat transfer and reversible).

The Carnot Cycle

The Carnot cycle is the most efficient power cycle and it is composed of four totally reversible processes:

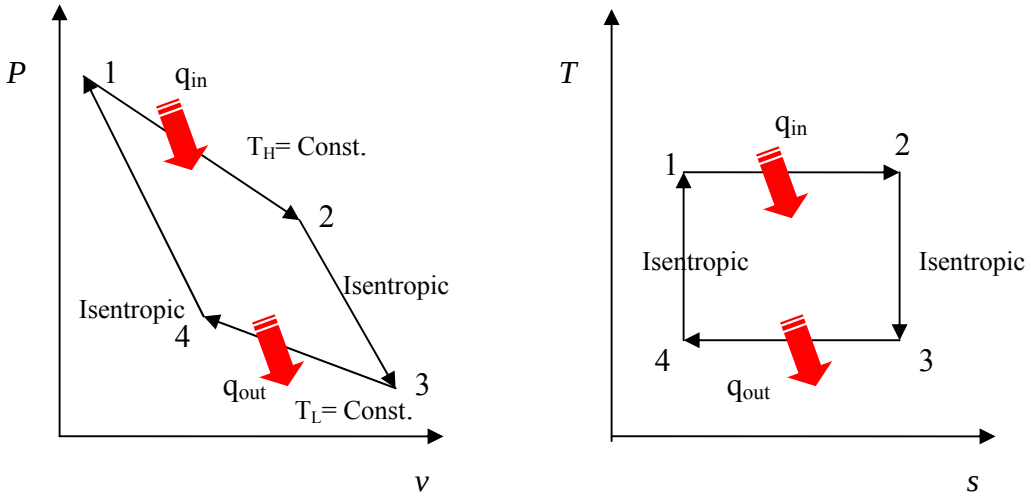


Fig. 2-2: P-v and T-s diagrams for the Carnot cycle (in a cylinder-piston).

Process 1-2, isothermal heat transfer (heat supply): heat is supplied at constant temperature T_H

Process 2-3, isentropic expansion (work output): air expands isentropically from the high pressure and temperature to the low pressure and temperature

Process 3-4, isothermal heat transfer (condenser or heat rejection): heat is rejected at constant temperature T_L

Process 4-1, isentropic compression (work in): the air compresses isentropically to the high pressure and temperature.

Thermal efficiency of the Carnot cycle can be calculated from:

$$\eta_{th,Carnot} = 1 - \frac{T_L}{T_H}$$

Carnot cycle is not limited to processes of a closed system taking place in a piston-cylinder assembly. Figure 2-3 shows the schematic and accompanying P-*v* diagram of a Carnot cycle executed by water steadily circulating through a simple vapor power plant.

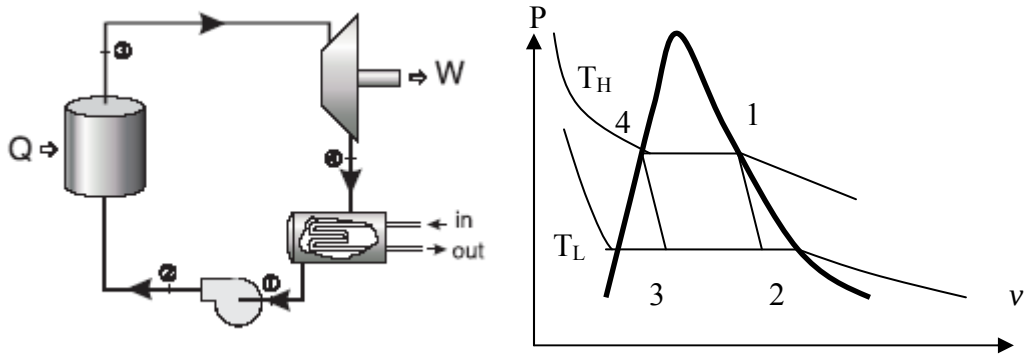


Fig. 2-2: Carnot vapor cycle.

1-2: The steam exiting the boiler expands adiabatically through the turbine and work is developed. The steam temperature decreases from T_H to T_L

2-3: Two-phase mixture flows through boiler where heat rejection occurs at constant temperature T_L.

3-4: The mixture (liquid-vapor) enters the pump (or compressor) and its pressure increases. This is an adiabatic process and requires input work.

4-1: The saturated liquid at high pressure enters the boiler, where a change of phase (from liquid to vapor) occurs at constant temperature T_H (isothermal heat transfer).

Carnot cycles are not encountered in reality since achieving isothermal heat transfer is very difficult (needs very large heat exchangers and long time). Also process 3-4 requires a pump or compressor that can handle two phase mixture (liquid and vapor) which is impractical.

The real value of the Carnot cycle comes from its being a standard against which the actual or ideal cycles can be compared.

Entropy

The second law leads to the definition of a new property called *entropy*.

The Clausius Inequality

The first law is simply an energy balance. However, the second law leads to an inequality; an *irreversible* process is less efficient than a reversible process. Another important inequality in thermodynamics is the *Clausius* inequality:

$$\oint \frac{\delta Q}{T} \leq 0$$

That is, the cyclic integral of δQ / T is always less than or equal to zero. This is valid for all cycles, reversible or irreversible.

For internally reversible cycles, it can be shown that:

$$\oint \frac{\delta Q}{T}\bigg|_{\text{int},rev} = 0$$

Entropy

The Clausius inequality forms the basis for the definition of a new property called entropy. As can be seen in the equation above, for an internally reversible process the cyclic integral of δQ / T is zero. A quantity whose cyclic integral is zero depends on the *state* only and not the process path, and thus it is a *property*.

Clausius in 1865 realized that he discovered a new property and he called it *entropy*:

$$dS = \left(\frac{\delta Q}{T}\right)_{\text{int},rev} \qquad \text{(kJ/K)}$$

Entropy per unit mass is designated by *s* (kJ/kg.K).

The entropy change of a system during a process can be calculated:

$$\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_{\text{int},rev} \qquad \text{(kJ/K)}$$

To perform this integral, one needs to know the relation between *Q* and *T* during the process.

Note that the cyclic integral of δQ / T will give us the entropy change *only if* the integration carried out along an internally reversible path between two states.

For irreversible processes, we may imagine a reversible process between the two states (initial and final) and calculate the entropy change (since entropy is a property).

The Increase of Entropy Principle

Entropy change of a closed system during an irreversible process is greater that the integral of δQ / T evaluated for the process. In the limiting case of a reversible process, they become equal.

$$dS \geq \frac{\delta Q}{T}$$

The entropy generated during a process is called *entropy generation,* and is denoted by S_{gen},

$$\Delta S = S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + S_{gen}$$

Note that the entropy generation S_{gen} is *always a positive quantity or zero* (reversible process). Its value depends on the process, thus it **is not a property** of a system.

The entropy of an isolated system during a process always increases, or in the limiting case of a reversible process remains constant (it never decreases). This is known as the increase of entropy principle.

The entropy change of a system or its surroundings can be negative; but entropy generation cannot.

$$S_{gen} = \begin{cases} > 0 & \text{irreversible process} \\ = 0 & \text{reversible process} \\ < 0 & \text{impossible process} \end{cases}$$

1- A process must proceeds in the direction that complies with the increase of entropy principle, $S_{gen} > 0$. A process that violates this principle is impossible.

2- Entropy is a non-conserved property, and there is no such thing as the conservation of entropy. Therefore, the entropy of universe is continuously increasing.

3- The performance of engineering systems is degraded by the presence of irreversibility. The entropy generation is a measure of the magnitudes of the irreversibilities present during the process.

Entropy Balance

Entropy is a measure of molecular disorder or randomness of a system, and the second law states that entropy can be created but it cannot be destroyed.

The increase of entropy principle is expressed as

Entropy change = Entropy transfer + Entropy generation

$$\Delta S_{system} = S_{transfer} + S_{gen}$$

This is called the entropy balance.

Entropy Change

The entropy balance is easier to apply that energy balance, since unlike energy (which has many forms such as heat and work) entropy has only one form. The entropy change for a system during a process is:

Entropy change = Entropy at final state - Entropy at initial state

$$\Delta S_{system} = S_{final} - S_{initial}$$

Therefore, the entropy change of a system is zero if the state of the system does not change during the process. For example *entropy change of steady flow devices* such as nozzles, compressors, turbines, pumps, and heat exchangers is *zero during steady operation*.

Mechanisms of Entropy Transfer

Entropy can be transferred to or from a system in two forms: heat transfer and mass flow. Thus, the entropy transfer for an adiabatic closed system is zero.

Heat Transfer: heat is a form of disorganized energy and some disorganization (entropy) will flow with heat. Heat rejection is the only way that the entropy of a fixed mass can be decreased. The ratio of the heat transfer Q/ T (absolute temperature) at a location is called entropy flow or entropy transfer

$$\text{Entropy transfer with heat } (\text{T} = \text{const.}) \qquad S_{heat} = \frac{Q}{T}$$

Since T (in Kelvin) is always positive, the direction of entropy transfer is the same of the direction of heat transfer.

When two systems are in contact, the entropy transfer from warmer system is equal to the entropy transfer to the colder system since the boundary has no thickness and occupies no volume.

Note that **work is entropy-free**, and no entropy is transferred with work.

Mass Flow: mass contains entropy as well as energy, both entropy and energy contents of a system are proportional to the mass. When a mass in the amount of *m* enters or leaves a system, entropy in the amount of *ms* (*s* is the specific entropy) accompanies it.

Entropy Balance for a Closed System

A closed system includes no mass flow across its boundaries, and the entropy change is simply the difference between the initial and final entropies of the system.

The entropy change of a closed system is due to the entropy transfer accompanying heat transfer and the entropy generation within the system boundaries:

Entropy change of the system = Entropy transfer with heat + Entropy generation

$$S_2 - S_1 = \sum \frac{Q_k}{T_k} + S_{gen}$$

Therefore, for an adiabatic closed system, we have:

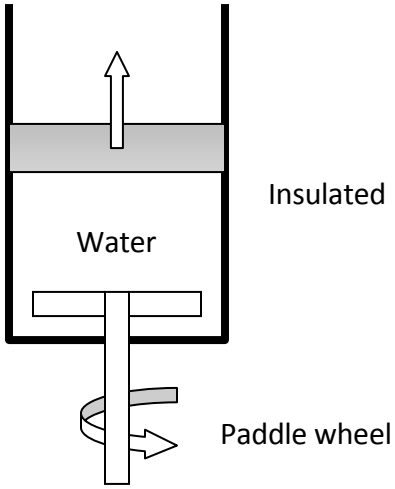
$$\Delta S_{adiabatic} = S_{gen}$$

For an internally reversible adiabatic process $\Delta S = 0$, because $S_{gen} = 0$.

The total entropy generated during a process can be determined by applying the entropy balance to an extended system that includes both the system and its immediate surroundings where external irreversibility might be occurring.

Example 1: Entropy balance for a closed system

Saturated liquid water at 100 C is contained in a piston-cylinder assembly. The water undergoes a process to the corresponding saturated vapor state, during which the piston moves freely in the cylinder. There is no heat transfer with the surroundings. If the change of state is brought about by the action of a paddle wheel, determine the network per unit mass, in kJ/kg, and the amount of entropy produced per unit mass, in kJ/kg.K.



Assumptions:

1- The water in the piston-cylinder assembly is a closed system.

2- There is no heat transfer with the surroundings.

3- The system is at an equilibrium state initially and finally. $\Delta PE = \Delta KE = 0$.

Solution

The network can be calculated by using the law:

$$\Delta U + \Delta KE + \Delta PE = Q - W$$

That is simplifies to: $\Delta U = - W$

On a unit mass basis, the energy balance becomes:

$$W / m = - (u_g - u_f)$$

From Table A-4,

$$W / m = - 2087.6 \text{ kJ/kg}$$

The negative sign indicates that the work input by the stirring is greater than the work done by the water as it expands.

Using an entropy balance, the amount of entropy produced can be found. Since there is no heat transfer,

$$\Delta S = \underbrace{\int_1^2 \left(\frac{\delta Q}{T} \right)}_{0} + S_{gen} = S_{gen}$$

On a unit mass basis, this becomes:

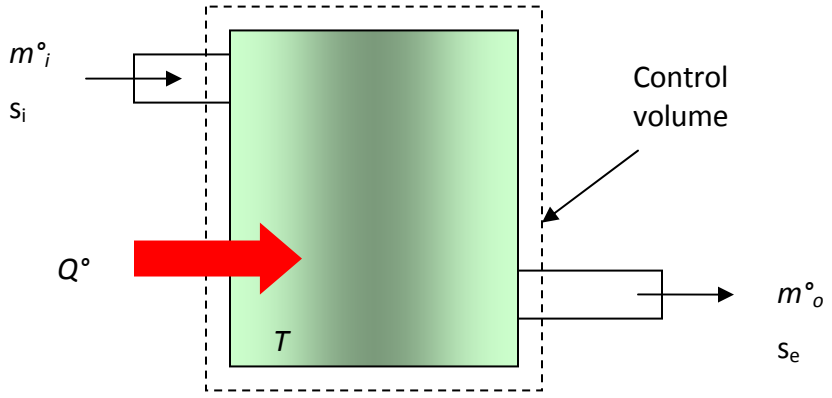
$$S_{gen} / m = s_g - s_f$$

Using Table A-4

$$S_{gen} / m = 6.048 \text{ kJ / kg.K}$$

Entropy Balance for a Control Volume

In addition to methods discussed for closed system, the entropy can be exchanged through *mass flows* across the boundaries of the control volume.



The entropy balance in the rate form for a control volume becomes:

$$\frac{dS_{CV}}{dt} = \sum \frac{\dot{Q}_k}{T_k} + \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \dot{S}_{gen,CV}$$

For a steady-state steady-flow process, it simplifies to:

$$\dot{S}_{gen,CV} = \sum \dot{m}_e s_e - \sum \dot{m}_i s_i - \sum \frac{\dot{Q}_k}{T_k}$$

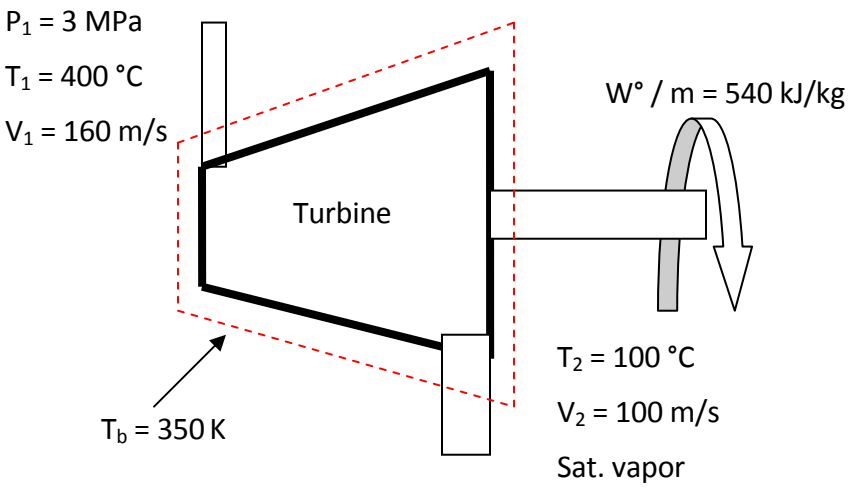
Example 2: Entropy balance for a CV

Steam enters a turbine with a pressure of 3 MPa, a temperature of 400 °C, and a velocity of 160 m/s. Saturated vapor at 100 °C exits with a velocity of 100 m/s. At steady-state, the turbine develops work equal to 540 kJ/kg. Heat transfer between the turbine and its surroundings occur at an average outer surface temperature of 350 K. Determine the rate at which entropy is produced within the turbine per kg of steam flowing, in kJ/kg.K. Neglect the change in potential energy between inlet and exit.

Assumptions:

1- Steady state operation in CV. ΔPE = 0.

2- Turbine outer surface is at a specified average temperature.



From the mass balance, we know that $\dot{m} = \dot{m}_1 = \dot{m}_2$

Since the process is steady-state, one can write:

$$0 = \sum \frac{\dot{Q}_k}{T_k} + \dot{m}(s_i - s_e) + \dot{S}_{gen,CV}$$

The heat transfer occurs at T_b = 350 K, the first term of the right hand side of the entropy balance reduces to $\dot{Q}/T_b$

$$\frac{\dot{S}_{gen,CV}}{\dot{m}} = -\frac{\dot{Q}_k}{\dot{m} T_k} + (s_2 - s_1)$$

We need to calculate the rate of heat transfer. The first law (energy balance) can be used to find the heat transfer rate. Combining the mass balance and the first law, one finds:

$$\frac{\dot{Q}_{CV}}{\dot{m}} = \frac{\dot{W}_{CV}}{\dot{m}} + (h_2 - h_1) + \left(\frac{V_2^2 - V_1^2}{2}\right)$$

From Table A-6, h_1 = 3230.9 kJ/kg, and From A-4 h_2 = 2676.1 kJ/kg. After substitution, and converting the units, one finds:

$$\frac{\dot{Q}_{CV}}{\dot{m}} = -22.6 \quad kJ/kg$$

From Table A-4, s_2 = 7.3549 kJ/kg.K and from Table A-6, s_1 = 6.9212 kJ/kg.K. Inserting values into the expression for entropy production:

$$\frac{\dot{S}_{gen,CV}}{\dot{m}} = -\frac{\dot{Q}_k}{\dot{m} T_k} + (s_2 - s_1) = 0.4983 \quad kJ/kg.K$$

Entropy

Entropy can be viewed as a measure of molecular disorder, or molecular randomness. As a system becomes more disordered, the positions of the molecules become less predictable and the entropy increases.

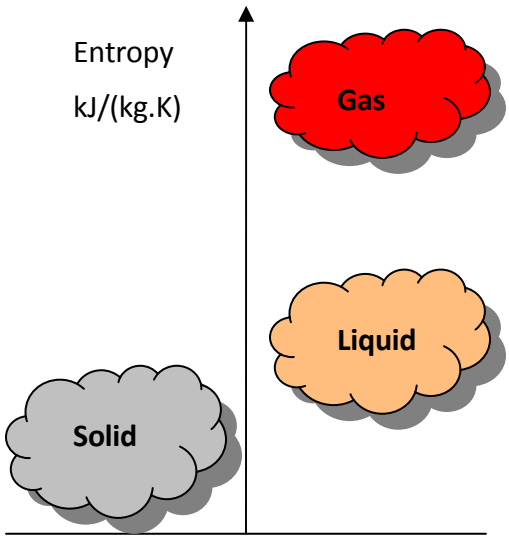


Fig. 1: Entropy of a substance (level of disorder) increases when it melts from solid phase to liquid. $S_{solid} < S_{liquid} < S_{gas}$

Some remarks:

Work is an organized form of energy, free of disorder or randomness, thus free of entropy. Therefore, there is *no entropy associated with energy transfer as work*.

The *quantity* of energy is always preserved during an actual process, based on the *first law*, but the *quality* is bound to decrease (*the second law*).

Processes can occur only in the direction of *increased overall entropy* or molecular disorder. Thus, the entire universe is getting more and more chaotic every day.

At *absolute zero* (0 K), molecules become completely motionless, this represents a state of ultimate molecular order (and *minimum energy*). Therefore, the entropy of a pure

crystalline substance at zero temperature is zero. That is because; there is *no uncertainty* about the state of the molecules at that instant. This statement is the **third law of thermodynamics**.

Since there is a reference for entropy (absolute zero), entropy is an absolute property. The entropy measured with respect to absolute zero is called absolute entropy.

The two diagrams used most extensively in the second-law analysis are the T-s and h-s diagrams. For an internally reversible process, one can write:

$$\delta Q_{\text{int},rev} = Tds \qquad (\text{kJ})$$

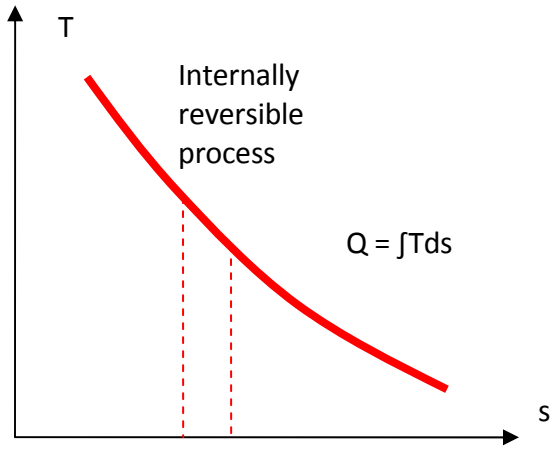


Fig. 2: On a T-s diagram, the area under an internally reversible process presents the heat transfer for the process.

For an internally reversible isothermal process, we have:

$$Q_{\text{int,rev}} = T_0\, ds$$

In a T-s diagram, an isentropic process is represented by a vertical line. An isentropic process is a process in which entropy remains constant. As a result an isentropic process involves no heat transfer. Therefore:

$$\text{Isentropic process } (s_2 = s_1) = \text{Reversible} + \text{Adiabatic}$$

Evaluation of Entropy Change

The differential form of the conservation of energy for a closed system (fixed mass) for an internally reversible process is:

$$\delta Q_{\text{int,rev}} - \delta W_{\text{int,rev}} = dU$$

where,

$$\delta Q_{\text{int,rev}} = TdS$$

$$\delta W_{\text{int,rev}} = PdV$$

Thus,

$$TdS = dU + PdV$$

or, per unit mass

$$Tds = du + Pdv$$

This is called the *first Gibbs equation*.

From the definition of enthalpy, h = u + Pv, one can find:

$$h = u + Pv \rightarrow dh = du + Pdv + vdP$$

Eliminating du from the first Gibbs equation, one finds the *second Gibbs equation*:

$$Tds = dh - vdP$$

Explicit relations for differential changes in entropy can be obtained from Gibbs equations:

$$ds = \frac{du}{T} + \frac{Pdv}{T}$$

$$ds = \frac{dh}{T} - \frac{vdP}{T}$$

To calculate the entropy change, we must know the relationship between du or dh and temperature.

Calculation of the Entropy for Saturated Mixture

Use Tables A-4 and A-5 to find s_f, s_g and/or s_{fg} for the following:

$$s = (1 - x)s_f + x\, s_g \quad \text{or} \quad s = s_f + x\, s_{fg}$$

Calculation of the Entropy for Superheated Vapor

Given two properties or the state, such as temperature and pressure, use Table A-6.

Calculation of the Entropy for Compressed Liquid

In the absence of compressed liquid data for a property $s \approx s_{f@T}$

Entropy Change of Solids and Liquids

Solids and liquids can be assumed as incompressible substances since their volumes remains essentially constant during a process. Thus, the first Gibbs equation becomes:

$$ds = \frac{du}{T} = \frac{cdT}{T}$$

$$s_2 - s_1 = \int_1^2 c(T)\frac{dT}{T}$$

Assuming an averaged value for specific heat, one obtains:

$$s_2 - s_1 = c_{ave} \ln \frac{T_2}{T_1}$$

Note that the entropy change of an incompressible substance is only a function of temperature. Therefore, for an *isentropic process* where $s_2 = s_1$, one can find:

$$T_2 = T_1$$

Entropy Change of Ideal Gas

The entropy change of an ideal gas can be obtained, by substituting $du = c_v\, dT$ and $P = RT/v$ into Gibbs equation.

$$ds = c_v \frac{dT}{T} + R\frac{dv}{v}$$

$$s_2 - s_1 = \int_1^2 c_v(T)\frac{dT}{T} + R\ln\frac{v_2}{v_1}$$

Assuming averaged values for specific heats, one obtains:

$$s_2 - s_1 = c_{v,ave} \ln \frac{T_2}{T_1} + R\ln\frac{v_2}{v_1} \qquad \frac{kJ}{kg.K}$$

$$s_2 - s_1 = c_{p,ave} \ln \frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} \qquad \frac{kJ}{kg.K}$$

For isentropic processes of ideal gases, the following relationships can be found by setting ds = 0,

$$\ln\frac{T_2}{T_1} = -\frac{R}{C_v}\ln\frac{v_2}{v_1}$$

$$\ln\frac{T_2}{T_1} = \ln\left(\frac{v_1}{v_2}\right)^{\frac{R}{C_v}} \quad or \quad \left(\frac{T_2}{T_1}\right) = \left(\frac{v_1}{v_2}\right)^{k-1} \quad \text{isentropic process}$$

Since $R = c_p - c_v$, $k = c_p / c_v$, and thus $R / c_v = k - 1$.

In a similar manner, one finds:

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} \quad \text{isentropic process}$$

$$\left(\frac{P_2}{P_1}\right) = \left(\frac{v_1}{v_2}\right)^{k} \quad \text{isentropic process}$$

These equations can be expressed in the following compact forms:

$$Tv^{k-1} = \text{constant}$$

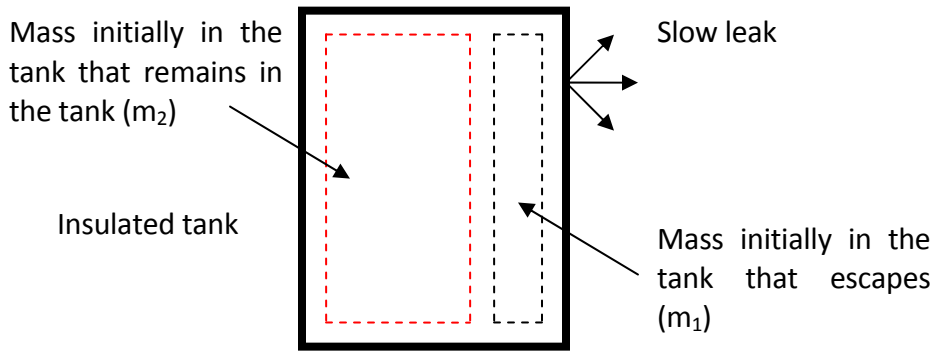
$$TP^{(1-k)/k} = \text{constant}$$

$$Pv^k = \text{constant}$$

The specific ratio *k*, varies with temperature, and in isentropic relations above an average *k* value should be used.

Example 3: Isentropic process of ideal gas

A rigid, well-insulated tank is filled initially with 5 kg of air at pressure 500 kPa and a temperature 500 K. A leak develops, and air slowly escapes until the pressure of the air remaining in the tank is 100 kPa. Using the ideal gas model, determine the amount of mass remaining in the tank and its temperature.



Assumptions:

1- As shown in the figure, the closed system is the mass initially in the tank that remains in the tank.

2- There is no significant heat transfer between the system and its surroundings.

3- Irreversibilities within the tank can be ignored as the air slowly escapes.

Solutions:

Using the ideal gas equation of state, the mass initially in the tank that remains in the tank at the end of process is:

$$\left.\begin{aligned} m_2 &= \frac{P_2 V}{RT_2} \\ m_1 &= \frac{P_1 V}{RT_1} \end{aligned}\right\} \rightarrow m_2 = \left(\frac{P_2}{P_1}\right)\left(\frac{T_1}{T_2}\right) m_1$$

Since the volume of the tank V remains constant during the process. We need to find the final temperature T_2. For the closed system under consideration (m_1), there are no irreversibilities, and no heat transfer. Accordingly, it is an *isentropic* process, and thus the isentropic relationships can be used:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} \qquad T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$

With a constant k = 1.4 for air, after substituting values, one finds:

$$T_2 = 315.55 \text{ K}$$

Finally, inserting values into the expression for system mass

$$m_2 = (100/500)\ (500/315.55)\ (5 \text{ kg}) = 1.58 \text{ kg}$$

Reversible Steady-Flow Work

The conservation of energy equation for a steady-flow device undergoing an internally reversible process can be expressed in differential form as

$$\delta q_{rev} - \delta w_{rev} = dh + d\text{ke} + d\text{pe}$$

But

$$\left.\begin{array}{l} \delta q_{rev} = Tds \\ Tds = dh - vdP \end{array}\right\} \rightarrow \delta q_{rev} = dh - vdP$$

Substituting into the relation above, after canceling dh, it yields,

$$-\delta w_{rev} = vdP + dke + dpe$$

Integrating, we find

$$w_{rev} = -\int_1^2 vdP - \Delta ke - \Delta pe \qquad (kJ/kg)$$

With negligible changes in potential and kinetic energies,

$$w_{rev} = -\int_1^2 vdP \qquad (kJ/kg)$$

From the above equation can be seen that, the larger the specific volume the larger the reversible produced or consumed work by the steady-flow device. Thus, every effort should be made to keep the specific volume of the flow as small as possible during a compression process to minimize the input work.

When the fluid is incompressible, the specific volume remains constant during the process, thus the above equation becomes:

$$\delta w_{rev} = v(P_1 - P_2) - \Delta ke - \Delta pe \quad \text{(kJ/kg)}$$

For a steady-state flow of a liquid through a device that involves no work interactions (such as nozzle or a pipe section), the work term is zero,

$$v(P_2 - P_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) = 0$$

This is known as *Bernoulli equation* in fluid mechanics.

Analysis of the application of ideal gas equation of state

YarongWang[1*] and PeirongWang[2]

[1] Baoding Electric Power VOC.&TECH. College, Baoding, Hebei, 071051, China
[2] Hebei Software Institute, Baoding, Hebei, 071000, China

Abstract. In nature, the molecules of real gas have a certain volume and have interaction force with each other. It is difficult to find the molecular motion law of real gas because of its complex properties. An ideal gas is an imaginary substance that does not exist in reality. Its molecules are elastic, non volume particles, and there is no interaction among them. This kind of gas is simple in nature and easy to be analyzed and calculated by simple mathematical relation. The introduction of the concept of ideal gas greatly simplifies the analysis of some thermodynamic problems.

1 Introduction

In thermal equipment, the energy transfer and conversion depend on the state change of working medium. The results of energy transfer and conversion are different due to different thermodynamic properties and state change processes of working medium. Therefore, in the study of conversion of heat and work, we must be familiar with the thermodynamic properties of common working medium. In nature, the molecules of real gas have volume, and there are some force interactions among molecules. When the specific volume of gas is not very large, these two factors have a certain impact on the molecular motion, which makes the molecular motion law complex. The volume of molecules and the interaction among molecules must be considered in engineering calculation. This makes the nature of the real gas more complex, the relationship of state parameters is more complex. In order to simplify the analysis and calculation in thermodynamics, the concept of ideal gas is put forward. It is considered that the molecules of ideal gas are elastic and non volume particles, and there are no any force interactions among molecules. In any equilibrium state, the expression describing the relationship among the basic state parameters of ideal gas is called the ideal gas equation of state.

2 Ideal gas equation of state.

2.1 state equation

The ideal gas follows Clapeyron equation. Clapeyron equation describes the relationship among the three state parameters of ideal gas in the same state, which is only applicable to ideal gas. It is also known as ideal gas equation of state. The equation has the following forms, where p, v, T represents pressure, specific volume and temperature, respectively:

$$pv = RT \tag{1}$$

$$pV_m = R_m T \tag{2}$$

$$pV = mRT \tag{3}$$

$$pV = nR_m T \tag{4}$$

Where
V_m— molar specific volume, m3/ kmol.

n— mole number, kmol.

R_m— the universal gas constant, it is the same for all kinds of ideal gases, J/(kmol•K).

In the standard condition, the molar specific volume of all ideal gases is 22.4 m^3/kmol.

Therefore, the universal gas constant can be calculated by the following formula [1]

$$\begin{aligned} R_m &= \frac{p_0 V_{m0}}{T_0} \\ &= \frac{1.01325\times10^5 \times 22.414}{273.15} \\ &= 8314 \quad J/(kmol\cdot K) \end{aligned} \tag{5}$$

R—gas constant,J/(kg•K).

It has nothing to do with the state of the gas, but it varies with the type of gas. That is to say, for the same gas, the value of gas constant is constant no matter in what state, but the constant value of different kinds of gas is different.

*Corresponding author's e-mail: bjoycg2008@163.com

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$$R_m = M \cdot R \tag{6}$$

Where

M—molar mass, kg/kmol.

Different gases have different molar masses. The thousand molar mass of a gas is numerically equal to its value of molecular weight.

The pressure in the equation is absolute pressure and the temperature is absolute pressure. The ideal gas equation of state is an approximate description of gas properties. When the gas density is large and the specific volume is very small, the relationship of p-v-T of real gas deviates from this equation significantly. Even at low density, the two are only roughly consistent. Only when the gas pressure is very low and the specific volume is very large, that is p→0, v→0, can the properties of the gas completely conform to the ideal gas equation of state [1]. Experiments have proved that when the pressure is low enough and the temperature is high enough, the specific volume of gas is large enough.

In practical calculation, many real gases can be treated as ideal gases with acceptable error. For example, the commonly used gases O_2, N_2, H_2, CO, CO_2 etc, as well as the air and flue gas composed of these gases, which are far away from the liquid area under normal pressure and temperature, can be treated as ideal gases. However, the steam generated in the boiler can not be treated as ideal gas because they are close to the liquid region. It must be specially pointed out that the water vapor contained in the flue gas and the atmosphere can still be studied as an ideal gas because of its small content. It can be seen that there is no obvious boundary between ideal gas and real gas. According to the allowable error range of engineering calculation, what kind of gas should be considered under what conditions should be determined [2].

2.2 Application examples

Examples: The pressure and temperature measured in the user pipe network of gas company to deliver natural gas to users are 200 MPa and 275 K respectively. If the pipeline diameter is 50mm and the natural gas flow rate is 5.5 m/s, please determine the mass flow rate and the volume flow rate under the standard state. The atmospheric pressure is 0.1 MPa [3].

Solution

$$\begin{aligned} p &= p_g + p_b \\ &= 200 + 0.1\times10^6 \\ &= 1.002\times10^5 \quad Pa \end{aligned} \tag{7}$$

$$T = 275 \quad K \tag{8}$$

$$p_0 = 1.01325\times10^5 \quad p_a \tag{9}$$

$$T_0 = 273 \quad K \tag{10}$$

$$\begin{aligned} R &= \frac{R_m}{M} \\ &= \frac{8314}{16} = 519.625 \quad J/(kg\cdot K) \end{aligned} \tag{11}$$

For the gas constants, we can calculate the gas constant in the above way. In addition, we can also refer to the relevant literature to make the values of molecular weight and the gas constant more accurate. As shown in Table 1[4]. According to different accuracy requirements, we can choose different methods.

$$\begin{aligned} R &= \frac{R_m}{M} \\ &= \frac{8314}{16.043} = 518.2 \quad J/(kg\cdot K) \end{aligned} \tag{12}$$

Table 1. Gas constants of several ideal-gases

Substance	Molecular formula	Molar mass	R [J/(kg • K)]	Substance	Molecular formula	Molar mass	R [J/(kg • K)]
Hydrogen	H_2	2.016	1124.0	Nitrogen	N2	28.013	296.8
Helium	He	4.003	2077.0	Carbon monoxide	CO	28.014	296.8
Methane	CH_4	16.043	518.2	Carbon dioxide	CO_2	44.010	288.9
Ammonia	NH_3	17.031	488.2	Oxygen	O_2	32.0	259.8
Water vapor	H_2O	18.015	461.5	Air		28.97	287.0

$$q_V = \frac{\pi D^2}{4} c = \frac{\pi \times 0.05^2}{4} \times 5.5 = 1.08 \times 10^{-2} \quad m^3/s \tag{13}$$

According to the four different forms of the ideal gas equation of state, four methods are used to calculate the mass flow rate.

Method 1

According to equation (1)

$$v = \frac{RT}{p} = \frac{519.625 \times 275}{1.002 \times 10^5} = 1.426 \quad m^3/kg \tag{14}$$

$$q_m = m = \frac{V}{v} = \frac{q_V}{v} = \frac{1.08 \times 10^{-2}}{1.426} = 7.57 \times 10^{-3} \quad kg/s \tag{15}$$

Method 2

According to equation (2)

$$v_m = \frac{R_m T}{p} = \frac{8314 \times 275}{1.002 \times 10^5} = 22.817 \quad m^3/kmol \tag{16}$$

$$v = \frac{v_m}{M} = \frac{22.817}{16} = 1.426 \quad m^3/kg \tag{17}$$

The mass flow rate is the mass of the gas flowing through the unit time:

$$q_m = m = \frac{V}{v} = \frac{q_V}{v} = \frac{1.08 \times 10^{-2}}{1.426} = 7.57 \times 10^{-3} \quad kg/s \tag{18}$$

Method 3

According to equation (3)

$$q_m = m = \frac{pV}{RT} = \frac{1.002 \times 10^5 \times 1.08 \times 10^{-2}}{519.625 \times 275} = 7.57 \times 10^{-3} \quad kg/s \tag{19}$$

Method 4

According to equation (4)

$$n = \frac{pV}{R_m T} = \frac{1.002 \times 10^5 \times 1.08 \times 10^{-2}}{8314 \times 275} = 4.733 \times 10^{-4} \quad kmol \tag{20}$$

$$q_m = m = nM = 4.733 \times 10^{-4} \times 16 = 7.57 \times 10^{-3} \quad kg/s \tag{21}$$

Although they are all not very complicated, it is obvious that the third form of equation, method 3, is the simplest for this example.

The volume flow rate is calculated as follows

Method 1

According to equation (3)

$$V_0 = \frac{mRT_0}{p_0} = \frac{7.57 \times 10^{-3} \times 519.625 \times 273}{1.01325 \times 10^5} = 1.06 \times 10^{-2} \quad m^3/s \tag{22}$$

The volume flow rate is the volume of gas flowing through the unit time:

$$q_{V_0} = V_0 = 1.06 \times 10^{-2} \quad m^3/s \tag{23}$$

Method 2

According to equation (3), under the condition of constant mass, there is

$$mR = \frac{pV}{T} = \frac{p_0 V_0}{T_0} \tag{24}$$

Therefore

$$V_0 = \frac{pV}{T} \cdot \frac{T_0}{p_0} = \frac{1.002 \times 10^5 \times 1.08 \times 10^{-2} \times 273}{275 \times 1.01325 \times 10^5} = 1.06 \times 10^{-2} \quad m^3/s \tag{25}$$

$$q_{V_0} = V_0 = 1.06\times10^{-2} \quad m^3/s \tag{26}$$

3 Conclusion

The assumption of ideal gas can meet the requirements of thermal engineering design and calculation, and the complexity and workload of calculation are reduced, so it plays a special role in engineering thermodynamics. Therefore, ideal gas and ideal gas equation of state occupy a special position in engineering thermodynamics. The equation of state of ideal gas is an equation describing the relationship between the basic state parameters of ideal gas. There are several forms of the equation. In the use of ideal gas equation of state, we should pay attention to: absolute pressure must be used instead of gauge pressure; absolute temperature must be used instead of centigrade temperature; the units of each physical quantity must be coordinated. Several forms of the ideal gas equation of state are interlinked. When we use it, we just need to choose the simplest form. Through the analysis of examples, we can intuitively see the characteristics of different forms of equations in the application. For some vocational students who have just studied the basic courses, they often have no choice when facing different forms of equations. This analysis process is helpful for them to compare the use characteristics of different forms of equations, so as to better use the equations to solve more complex practical problems.

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Refrigeration Cycle

Heat flows in direction of decreasing temperature, i.e., from high-temperature to low temperature regions. The transfer of heat from a low-temperature to high-temperature requires a *refrigerator* and/or *heat pump*.

Refrigerators and heat pumps are essentially the same device; they only differ in their objectives.

The performance of refrigerators and heat pumps is expressed in terms of coefficient of performance (COP):

$$COP_R = \frac{Q_L}{W_{net,in}}$$

$$COP_{HP} = \frac{Q_H}{W_{net,in}}$$

The Reversed Carnot Cycle

Reversing the Carnot cycle does reverse the directions of heat and work interactions. A refrigerator or heat pump that operates on the reversed Carnot cycle is called a *Carnot refrigerator* or a *Carnot heat pump*.

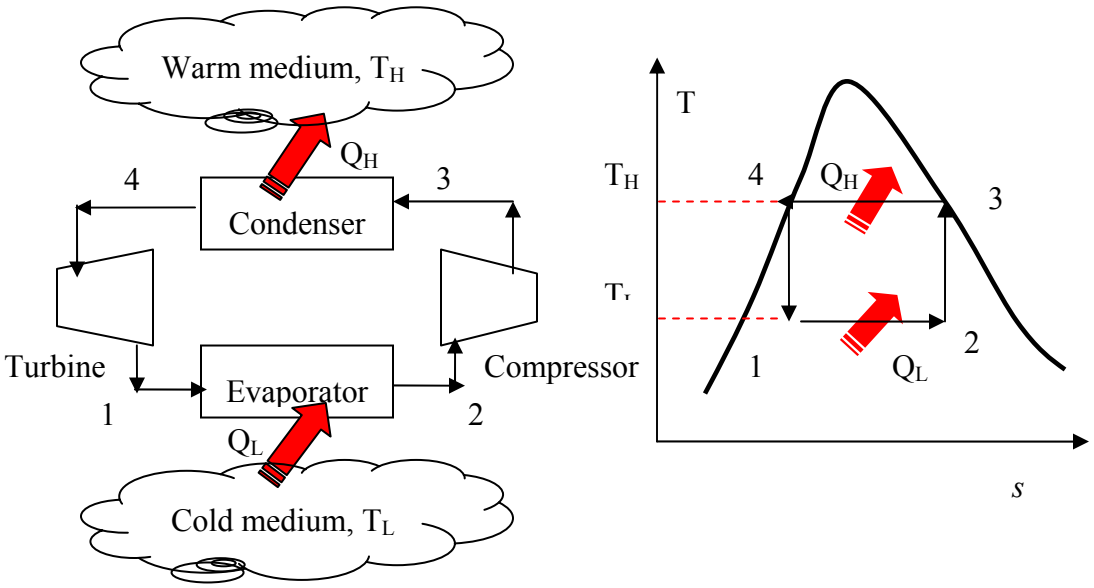


Fig. 5-1: T-s diagram and major components for Carnot refrigerator.

The reversed Carnot cycle is the most efficient refrigeration cycle operating between two specified temperature levels. It sets the highest theoretical COP. The coefficient of performance for Carnot refrigerators and heat pumps are:

$$COP_{Ref,Carnot} = \frac{1}{T_H / T_L - 1} \qquad COP_{HP,Carnot} = \frac{1}{1 - T_L / T_H}$$

The Carnot cycle cannot be approximated in an actual cycle, because:

1- executing Carnot cycle requires a compressor that can handle two-phases

2- also process 4-1 involves expansion of two-phase flow in a turbine.

The Ideal Vapor-Compression Refrigeration Cycle

The vapor-compression refrigeration is the most widely used cycle for refrigerators, air-conditioners, and heat pumps.

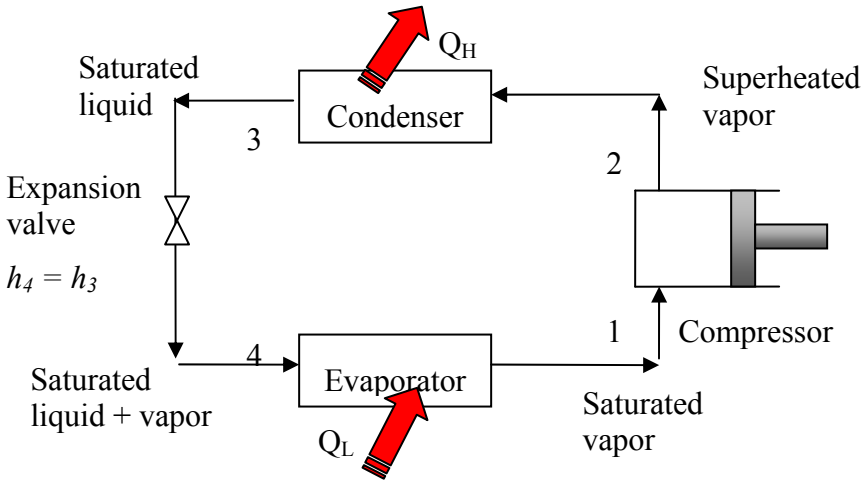


Fig. 5-2: Schematic for ideal vapor-compression refrigeration cycle.

Assumptions for ideal vapor-compression cycle:

- irreversibilities within the evaporator, condenser and compressor are ignored
- no frictional pressure drops
- refrigerant flows at constant pressure through the two heat exchangers (evaporator and condenser)
- heat losses to the surroundings are ignored
- compression process is isentropic

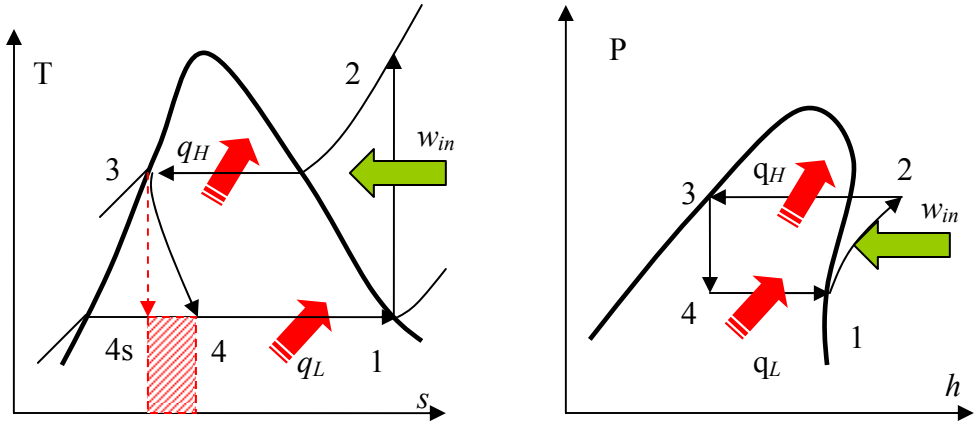


Fig. 5-3: *T-s* and *P-h* diagrams for an ideal vapor-compression refrigeration cycle.

1-2: A reversible, adiabatic (isentropic) compression of the refrigerant. The saturated vapor at state 1 is superheated to state 2.

$$w_c = h_2 - h_1$$

2-3: An internally, reversible, constant pressure heat rejection in which the working substance is de-superheated and then condensed to a saturated liquid at 3. During this process, the working substance rejects most of its energy to the condenser cooling water.

$$q_H = h_2 - h_3$$

3-4: An irreversible throttling process in which the temperature and pressure decrease at constant enthalpy. The refrigerant enters the evaporator at state 4 as a low-quality saturated mixture.

$$h_3 = h_4$$

4-1: An internally, reversible, constant pressure heat interaction in which the refrigerant (two-phase mixture) is evaporated to a saturated vapor at state point 1. The latent enthalpy necessary for evaporation is supplied by the refrigerated space surrounding the evaporator. The amount of heat transferred to the working fluid in the evaporator is called the refrigeration load.

$$q_L = h_1 - h_4$$

Notes:

The ideal compression refrigeration cycle is **not an internally reversible** cycle, since it involves throttling which is an irreversible process.

If the expansion valve (throttling device) were replaced by an isentropic turbine, the refrigerant would enter the evaporator at state 4s. As a result the refrigeration capacity would increase (area under 4-4s) and the net work input would decrease (turbine will

produce some work). However; replacing the expansion valve by a turbine is not practical due to the added cost and complexity.

The COP improves by 2 to 4% for each °C the evaporating temperature is raised or the condensing temperature is lowered.

Actual Vapor-Compression Refrigeration Cycle

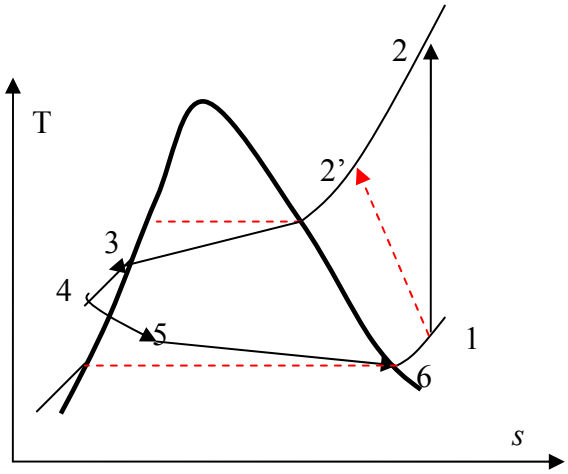


Fig. 5-4: T-s diagram for actual vapor-compression cycle.

Most of the differences between the ideal and the actual cycles are because of the irreversibilities in various components which are:

1-In practice, the refrigerant enters the compressor at state 1, slightly superheated vapor, instead of saturated vapor in the ideal cycle.

2- The suction line (the line connecting the evaporator to the compressor) is very long. Thus pressure drop and heat transfer to the surroundings can be significant, process 6-1.

3- The compressor is not internally reversible in practice, which increase entropy. However, using a multi-stage compressor with intercooler, or cooling the refrigerant during the compression process, will result in lower entropy, state 2'.

4- In reality, the refrigerant leaves condenser as sub-cooled liquid. The sub-cooling process is shown by 3-4 in Fig. 5-4. Sub-cooling increases the cooling capacity and will prevent entering any vapor (bubbles) to the expansion valve.

5- Heat rejection and addition in the condenser and evaporator do not occur in constant pressure (and temperature) as a result of pressure drop in the refrigerant.

Selecting the Right Refrigerant

When designing a refrigeration system, there are several refrigerants from which to choose. The right choice of refrigerant depends on the situation at hand. The most common refrigerants are: R-11, R-12, R-22, R-134a, and R-502.

R12: CCl_2F_2 dichlorofluoromethane, used for refrigeration systems at higher temperature levels- typically, water chillers and air conditioning (banned due to ozone layer effects)

R22: $CHClF_2$ has less chlorine, a little better for the environment than R12 - used for lower temperature applications

R134a: CFH_2CF_3 tetrafluorethane - no chlorine- went into production in 1991- replacement for R12

Ammonia NH_3: corrosive and toxic- used in absorption systems-cheap- high COP

R744: CO_2 behaves in the supercritical region- low efficiency

R290: propane combustible.

Many factors need to be considered when choosing a refrigerant:

ozone depletion potential ,global warming potential:	CFC (chlorofluorocarbons) refrigerants allow more ultraviolet radiation into the earth's atmosphere by destroying the protective ozone layer and thus contributing to the greenhouse effect that causes the global warming. As a result the use of some CFCs (e.g. R-11, R-12, and R-115) are banned by international treaties.
combustibility:	all hydro-carbon fuels, such as propane
leak detectability:	the refrigerant saturated pressure at the evaporator should be above P_{atm}.
thermal factors:	1- the heat of vaporization of the refrigerant should be high. The higher h_{fg}, the greater the refrigerating effect per *kg* of fluid circulated.
thermal factors cont'd.:	2- the specific heat of the refrigerant should be low. The lower the specific heat, the less heat it will pick up for a given change in temperature during the throttling or in flow through the piping, and consequently the greater the refrigerating effect per *kg* of refrigerant
	3- the specific volume of the refrigerant should be low to minimize the work required per *kg* of refrigerant circulated since evaporation and condenser temperatures are fixed by the temperatures of the surroundings
	4- selection is based on operating pressures in the evaporator and the condenser.

Other desirable characteristics of refrigerant are: non-flammable, being chemically stable, and be available at low cost.

Cascade Refrigeration Cycle

Systems that have 2 (or more) refrigeration cycles operating in series.

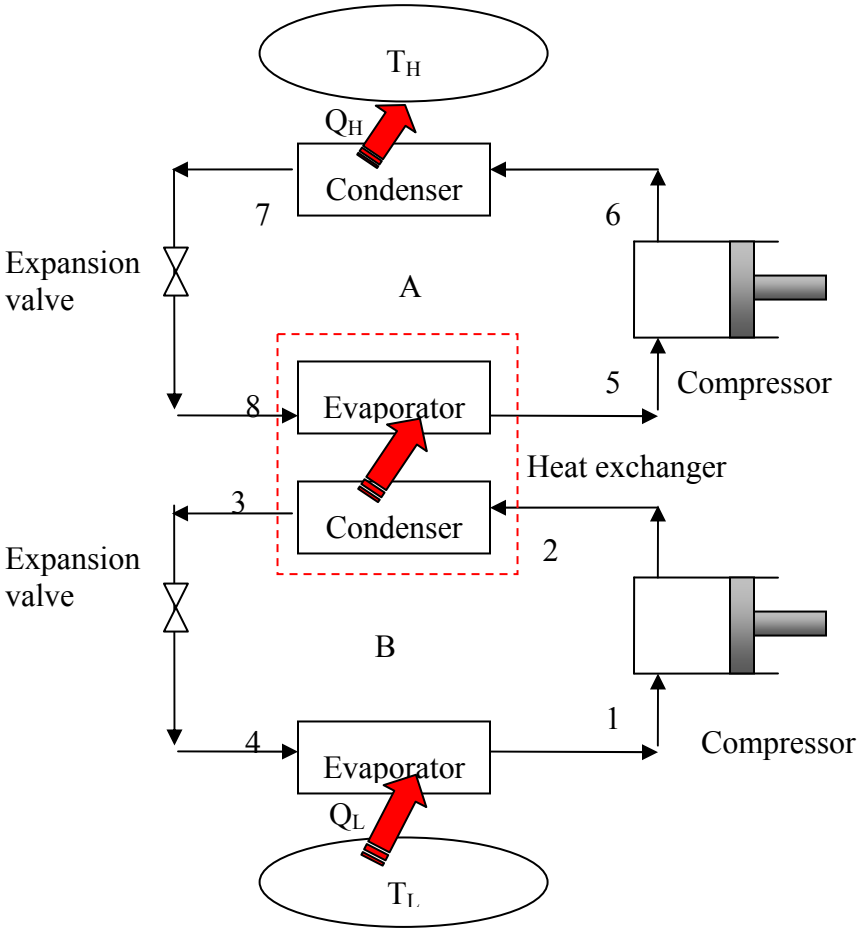


Fig.5-5: A 2-stage cascade refrigeration cycle.

Cascade cycle is used where a very wide range of temperature between T_L and T_H is required. As shown in Fig. 5-5, the condenser for the low temperature refrigerator is used as the evaporator for the high temperature refrigerator.

Cascading improves the COP of a refrigeration cycle. Moreover, the refrigerants can be selected to have reasonable evaporator and condenser pressures in the two or more temperature ranges.

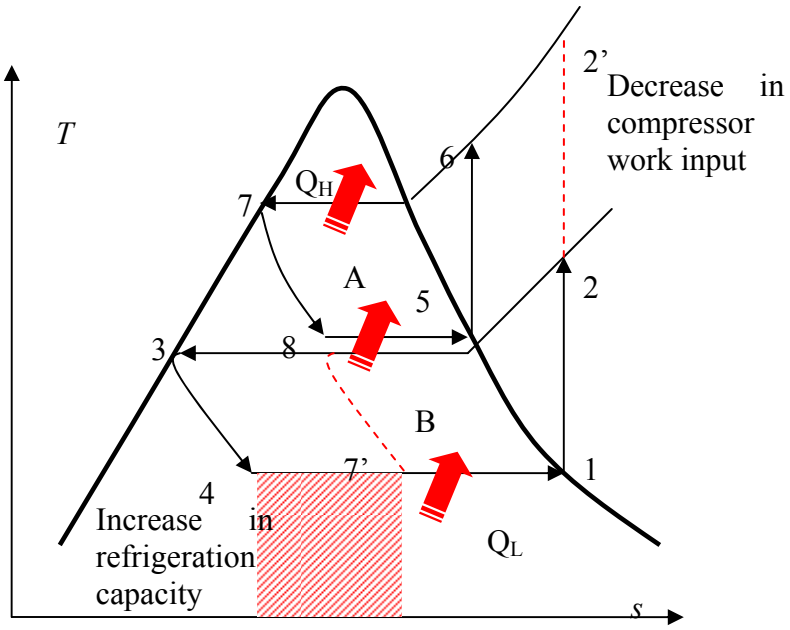


Fig. 5-6: T-s diagram for 2-stage cascade system.

The two cycles are connected through the heat exchanger in the middle, which serves as evaporator (cycle A) and condenser (cycle B). One can write:

$$\dot{m}_A(h_5 - h_8) = \dot{m}_B(h_2 - h_3)$$

$$COP_{R,cascade} = \frac{\dot{m}_B(h_1 - h_4)}{\dot{m}_A(h_6 - h_5) + \dot{m}_B(h_2 - h_1)}$$

Figure 5-6 shows the increase in refrigeration capacity (area under 4-7') and decrease in compressor work (2-2'-6-5).

Example 5-1: A two-stage Refrigeration cycle

Consider a two-stage cascade refrigeration system operating between pressure limits of 0.8 and 0.14 MPa. Each stage operates on an ideal vapor-compression refrigeration cycle with refrigerant R-134a as working fluid. Heat rejection from the lower cycle to the upper cycle takes place in an adiabatic counter flow heat exchanger where both streams enter at about 0.32 MPa. If the mass flow rate of the refrigerant through the upper cycle is 0.05 kg/s, determine:

a) the mass flow rate of the refrigerant through the lower cycle

b) the rate of heat removal from the refrigerated space and the power input to the compressor

c) COP.

Assumptions:

1) Steady operation
2) ΔKE=ΔPE=0
3) Adiabatic heat exchanger.

Figure 5-6 shows the *T-s* diagram for the cascade cycle. Enthalpies for all 8 states of refrigerant R-134a can be read off Tables A-12 and A-13.

Writing energy balance for the heat exchanger, the mass flow rate of the refrigerant through the lower cycle can be found:

$$\dot{m}_A h_5 + \dot{m}_B h_3 = \dot{m}_A h_8 + \dot{m}_B h_2$$

$$\dot{m}_A (h_5 - h_8) = \dot{m}_B (h_2 - h_3)$$

$$0.05 kg/s (251.88 - 95.47) kJ/kg = \dot{m}_B (255.93 - 55.16) kJ/kg$$

$$\dot{m}_B = 0.039 kg/s$$

b) the heat removal by the cascade cycle can be determined from:

$$\dot{Q}_L = \dot{m}_B (h_1 - h_4) = 0.0390 kg/s (239.16 - 55.16) kJ/kg = 7.18 kW$$

$$\dot{W}_{in} = \dot{W}_{Comp1,in} + \dot{W}_{Comp2,in} = \dot{m}_A (h_6 - h_5) + \dot{m}_B (h_2 - h_1) = 1.61 kW$$

c) the COP of the cycle will be:

$$COP_R = \frac{\dot{Q}_L}{\dot{W}_{net,in}} = \frac{7.18 kW}{1.61 kW} = 4.46$$